R.N. BASE

The Association
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Influence Lines for Statically Determinate Structures.

By D. WILSON, B.Sc. (Eng.).

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INFLUENCE LINES FOR STATICALLY DETERMINATE STRUCTURES.

By D. Wilson, B.Sc. (Eng.).

I.—GENERAL INTRODUCTION.

When a beam is subjected to a given fixed loading, then the shear force and bending moment diagrams are readily drawn. Also when a frame is subjected to a given fixed loading, the forces in the individual members are readily obtained. However, if the given system of loading was not fixed, then to find the maximum values of shear force, bending moment, or the forces in the individual members, a great number of diagrams and calculations would have to be made with the loads in different positions, until the maximum values were obtained. Even then the values may not be very accurate. Therefore it can be seen that the ordinary methods of solution for static loading only give the values of shear force, etc., for one position of the load.

An influence line will give the value of shear force, etc., for one point on the beam or one member of the frame for any position of the load. Influence lines are actually graphs (nearly always linear) which are very easy to draw, and show how the shear force, bending moment, or force in a member of a frame varies as a load rolls across the span.

Influence lines are always drawn for a unit of a 1 ton rolling load which crosses the whole span of the beam or frame. The application of a series of wheel loads which may come from road or rail traffic on to the frame, are best seen from worked out examples which are given as the different influence lines are developed.

In all the worked examples which are shown, it will be noticed that only one beam or girder is considered in the calculation, and that the systems of rolling loads are applied to this one girder only.

This has been adopted for simplicity, and it assumes that the rolling loads used will be a single set of wheel loads if there are two girders per track, or a combined set of axle loads if the bridge under consideration carries several tracks.

The systems of rolling loads are always assumed to move over the span from either end of the bridge and with either end of the load system leading, so that the greatest value for reaction, shear force, bending moment or force in a member of a framed girder is obtained for design purposes.

In the latter part of this pamphlet, only the influence lines will be drawn, it being assumed that the wheel loads can now be applied to give the maximum values of shear force, bending moment, etc., as described in the first part of the pamphlet.

In all the built-up girder examples the "Method of Sections" has been adopted to find the forces in the members concerned.

II.—INFLUENCE LINES FOR REACTIONS.

Figure 1 (a) shows a simply supported beam AB of span l, which carries a 1 ton rolling load on its span. Let the reactions at A and B be R_A and R_B respectively. When the 1 ton load is at a distance x from the support A, then we have :—

$$R_B = (x/l) \qquad \text{and } R_A = (l-x/l)$$

When the 1 ton load is at the reaction A then $R_A = 1{\cdot}0$ since $x = o$, and when the load is at the reaction B then $R_A = o$ since $x = l$. It can be seen from the equation for R_A that as the distance x varies from o to l, then the value of R_A varies linearly from $1{\cdot}0$ to o. Therefore the influence line for the reaction at A $(=R_A)$ is the straight line as shown in Fig. 1 (b), and the height of the influence line under the load is given by $(l-x/l)$ using similar triangles, which is the value of the reaction R_A for the load as shown. Therefore, the reaction R_A due to a 1 ton load at any point on the span is obtained by reading the height of the influence line directly under the load.

Now, when the 1 ton load is at the reaction A, then $R_B = o$, and when the 1 ton load is at the reaction B then $R_B = 1{\cdot}0$. Then using a similar argument as that used to derive the influence for reaction at A, we find that the influence line for the reaction at B is also linear, varying from o at support A to $1{\cdot}0$ at support B as shown in Fig. 1 (c). The height of this influence line under the 1 ton load is seen to be (x/l) from similar triangles, and the reaction R_B due to a 1 ton load at any part of the span is obtained by reading the height of the influence line directly under the load.

It is usual to combine the influence lines for the two reactions as shown in Fig. 1 (d).

To Draw the Influence Lines for Reactions.

Sketch the beam and project the two reactions vertically downwards. Draw a horizontal line to cut these two vertical lines to act as a zero line.

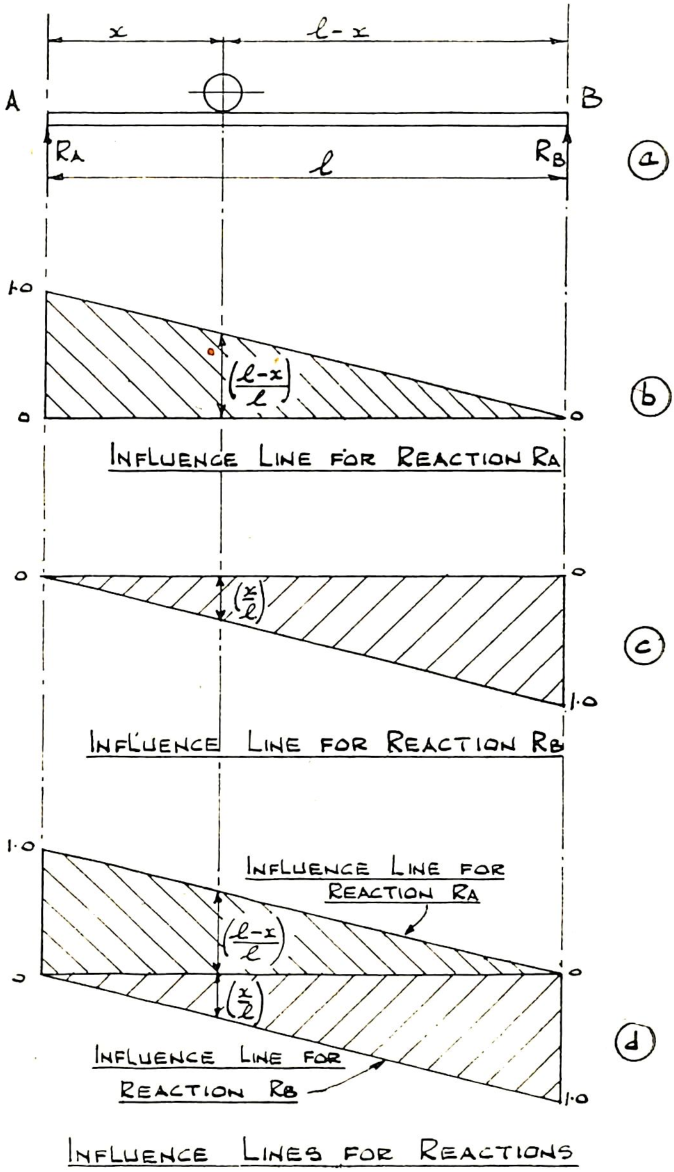

x
ℓ - x
A
B
RA
RB
ℓ
a
1·0
(ℓ-x / ℓ)
0
0
b
INFLUENCE LINE FOR REACTION RA
0
0
(x / ℓ)
1·0
c
INFLUENCE LINE FOR REACTION RB
1·0
INFLUENCE LINE FOR REACTION RA
(ℓ-x / ℓ)
0
0
(x / ℓ)
d
INFLUENCE LINE FOR REACTION RB
1·0
INFLUENCE LINES FOR REACTIONS

Fig. 1.

Under the left-hand reaction erect an ordinate of 1·0 *upwards* and join the top of this ordinate to the zero line at the right-hand reaction so that a triangle is formed above the zero line. This is the influence line for the left-hand reaction.

Under the right-hand reaction draw an ordinate of 1·0 *downwards* and join the bottom of this ordinate to the zero line at the left-hand reaction, so that a triangle is formed below the zero line. This is the influence line for the right-hand reaction.

The final diagram will be composed of two triangles which form a parallelogram as shown in Fig. 1 (d).

Example 1.—The system of wheel loads shown in Fig. 2 (a) rolls across a simply supported girder of 40 feet span. Calculate the maximum values of the reactions.

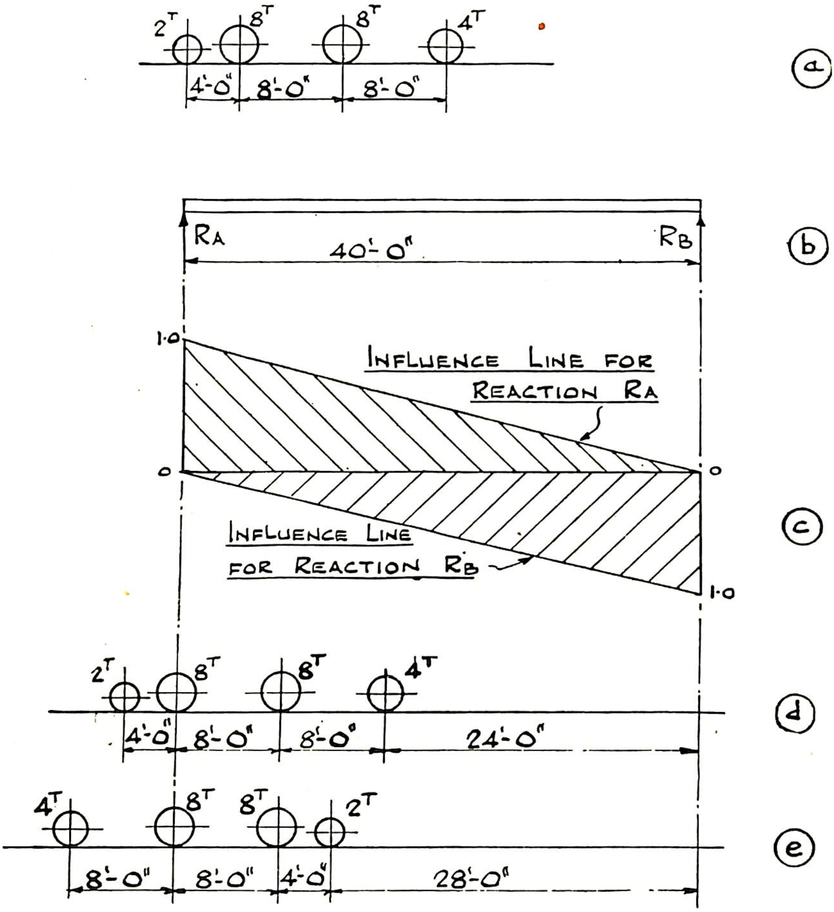


Fig. 2.

The influence lines for the reactions are drawn in Fig. 2 (c) as described above. Since the wheel load system may cross the girder in either direction with either the 2 ton or the 4 ton load leading, the maximum values of the two reactions, R_A and R_B will be the same. Therefore only the reaction R_A will be considered.

The maximum value of this reaction will always occur with one of the loads directly over the reaction (usually the heaviest load) but the actual maximum can only be obtained by trying the wheel load system in various positions. With a little practise it usually becomes quite obvious as to which position of the loads will give the greatest value for the reaction.

In this example the maximum value of reaction R_A will occur with the loading shown in one of the positions, Fig. 2 (d) and (e).

Now, if a 1 ton load at any point on a beam gives rise to a reaction W, then a 10 ton load at the same point will give rise to a reaction 10.W, etc. Also, when a series of loads is carried on a beam, the total reaction is the sum of the reactions due to each separate load.

Then, for the loading shown in Fig. 2 (d),

Reaction $R_A = (8 \times 1{\cdot}0) + (8 \times 1{\cdot}0 \times 32/40) + (4 \times 1{\cdot}0 \times 24/40)$
(using similar triangles)

$= 8{\cdot}0 + 6{\cdot}4 + 2{\cdot}4 =$ **16·8 tons.**

and for the loading shown in Fig. 2 (e)

Reaction $R_A = (8 \times 1{\cdot}0) + (8 \times 1{\cdot}0 \times 32/40) + (2 \times 1{\cdot}0 \times 28/40)$

$= 8{\cdot}0 + 6{\cdot}4 + 1{\cdot}4 =$ **15·8 tons.**

Therefore the maximum values of the reactions are 16·8 tons.

Example 2.—Two cranes run along a gantry which is a series of simply supported 50 feet spans. The maximum wheel loadings of the two cranes when running buffer to buffer (including a suitable allowance for shock loading) are given in Fig. 3 (a). Find the greatest reaction on to any one column.

AB and BC are taken as any two of the 50 feet spans of the girder in Fig. 3 (b), and the influence lines for the reaction at B are drawn for each of the simply supported spans in Fig. 3 (c). Then the maximum value of the reaction at B will occur when the wheel load system is in one of the positions shown in Figs. 3 (d) and 3 (e).

For the position shown in Fig. 3 (d),

$$\begin{aligned}\text{Reaction at B} &= (46 \times 1{\cdot}0) + (46 \times 1{\cdot}0 \times 45/50) + (40 \times 1{\cdot}0 \times 27/50) \\ &\qquad + (40 \times 1{\cdot}0 \times 22/50) \\ &\quad + (40 \times 1{\cdot}0 \times 41/50) + (40 \times 1{\cdot}0 \times 36/50) \\ &\qquad + (46 \times 1{\cdot}0 \times 18/50) + (46 \times 1{\cdot}0 \times 13/50) \\ &= \textbf{216·8 tons.}\end{aligned}$$

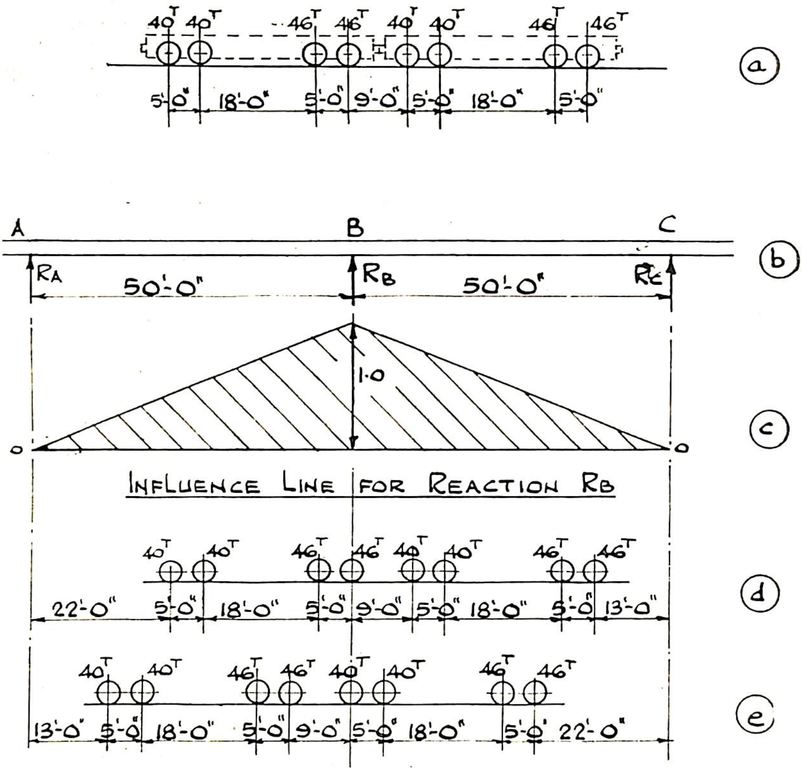


Fig. 3.

For the position shown in Fig. 3 (e),

$$\begin{aligned}\text{Reaction at B} &= (40 \times 1{\cdot}0) + (40 \times 1{\cdot}0 \times 45/50) + (46 \times 1{\cdot}0 \times 27/50)\\ &\quad + (46 \times 1{\cdot}0 \times 22/50) + (46 \times 1{\cdot}0 \times 41/50) + (46 \times 1{\cdot}0 \times 36/50)\\ &\quad + (40 \times 1{\cdot}0 \times 18/50) + (40 \times 1{\cdot}0 \times 13/50)\\ &= \mathbf{216{\cdot}8\ tons.}\end{aligned}$$

Therefore the maximum value of the reaction on to *any* one column is 216·8 tons, and occurs when the wheel loads are in either of the positions shown in Figs. 3 (d) and 3 (e).

III.—INFLUENCE LINES FOR SHEAR FORCE.

In Fig. 4 (a), AB is a beam of span l and C is the point in the span for which the influence line is required. The reactions at the supports A and B are represented by R_A and R_B respectively.

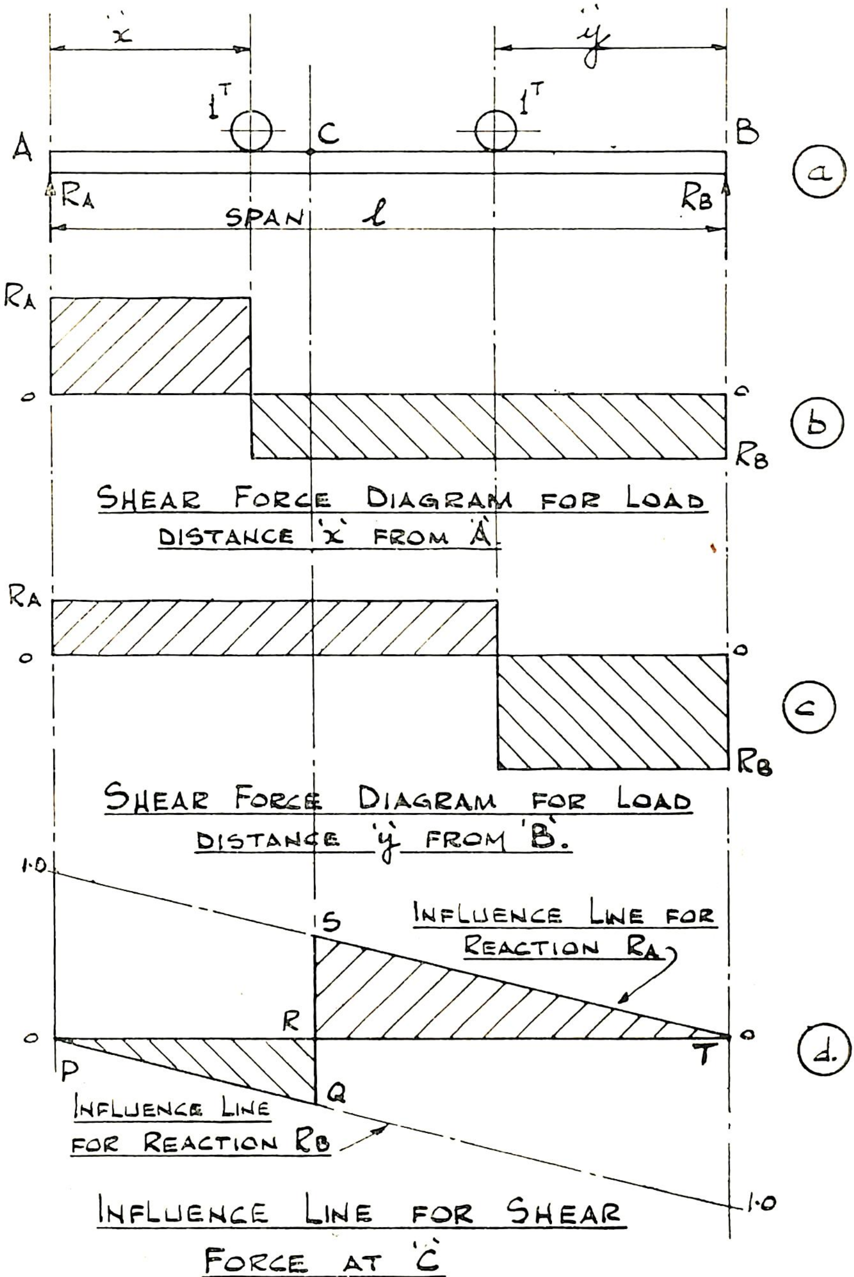

'x'
'y'
1T
1T
A
C
B
(a)
RA
RB
SPAN l
RA
0
0
RB
(b)
SHEAR FORCE DIAGRAM FOR LOAD
DISTANCE 'x' FROM 'A'
RA
0
0
(c)
RB
SHEAR FORCE DIAGRAM FOR LOAD
DISTANCE 'y' FROM 'B'.
1·0
S
INFLUENCE LINE FOR
REACTION RA
R
0
0
(d.)
P
T
Q
INFLUENCE LINE
FOR REACTION RB
1·0
INFLUENCE LINE FOR SHEAR
FORCE AT 'C'

Fig. 4.

Now if the 1 ton rolling load is between A and C at a distance x from A, then the shear force diagram is as shown in Fig. 4 (b), and the shear force at C is the same as the reaction R_B. This is always true when the load lies between A and C. Therefore, when the load is between A and C, the influence line for shear force at C is the same as the influence line for the reaction R_B.

Similarly if the 1 ton rolling load is between B and C distance y from B, the shear force diagram is as shown in Fig. 4 (c), and, for this case, the influence line for shear at C is the same as the influence line for the reaction R_A.

In Fig. 4 (d) the influence lines for the reactions R_B and R_A have been drawn, and a vertical cut made directly below the point C so that the two shaded triangles are formed. Then the line PQRST which encloses these two triangles is the influence line for shear force at the point C.

To Draw the Influence Line for Shear at any Point in a Beam.

Draw the beam, and then, faintly draw the influence lines for reactions. Make a vertical cut through the influence lines below the point considered and join to the base line at the supports to form *two triangles*. Then the line enclosing these triangles is the required influence line.

Note.—If the influence line diagram above the base line is taken as positive, then that below the base line must be taken as negative and *vice-versa*.

Example 3.—A girder of 40 feet span may be subjected to rolling loads of 2 tons, 10 tons and 6 tons in line, each pair of wheels being 6 feet apart, or to an equivalent uniformly distributed live load of $\frac{1}{2}$ ton per foot run longer than the span. Find the maximum shear force at a quarter span point.

Let P be the quarter span point as shown in Fig. 5 (a). Then the influence line for the shear force at point P has been drawn in Fig. 5 (b) as described above. The maximum shear force at P due to the rolling loads will occur when the loads are as shown in Fig. 5 (c).

Therefore shear force at P due to rolling loads

$$= (10 \times 0{\cdot}75) + (6 \times 0{\cdot}75 \times 24/30) - (0{\cdot}25 \times 2 \times 4/10) = 10{\cdot}9 \text{ tons}$$

To obtain the maximum shear force at P due to the uniformly distributed load, consider the elemental length δx of the load at a distance x from the right-hand support. The total load acting over the length δx is $(\frac{1}{2}\ \delta x)$ tons, the height of the influence line at this point being $(x/40)$. Then the shear force at P due to this elemental load $= (\frac{1}{2}\ \delta x\ x/40) = x.\delta x/80$ tons.

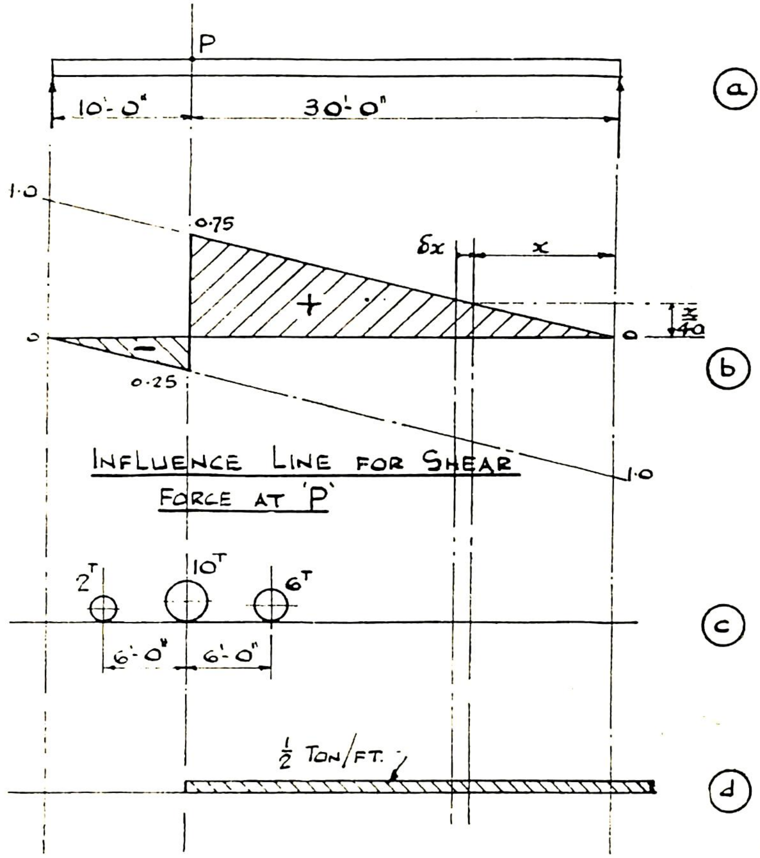


Fig. 5.

Then integrating for values of x between 0 and 30 feet we have the shear force at P due to the loading position shown in Fig. 5 (d)

$$\text{is} \int_0^{30} \frac{x}{80}\,dx = \left[\frac{x^2}{160}\right]_0^{30} = \frac{900}{160} = 5{\cdot}6 \text{ tons.}$$

This value is the same as that obtained by multiplying the area under the influence line covered by the load by the rate of loading :—

Shear force at P $=$ (area under influence line) $\times$ (load per foot run)
$= (\tfrac{1}{2} \times 30 \times 0{\cdot}75) \times (\tfrac{1}{2}) = 90/16 = 5{\cdot}6$ tons.

This is true for all linear influence lines when uniformly distributed loads are applied.

Therefore the maximum shear force at a quarter span point occurs when the rolling loads are acting and $=$ **10·9 tons.**

IV.—INFLUENCE LINES FOR BENDING MOMENT.

In Fig. 6 (a) the beam AB has a span l, the reactions at A and B being R_A and R_B respectively. The point C which divides the span AB in the ratio $a : b$, is the point for which the influence line is required.

Consider the 1 ton rolling load between A and C at a distance of x from A. Then $R_B = x/l$ and the bending moment at C $= x.b/l$.

This is a linear equation in x which varies from zero when $x = o$ up to a maximum value of (ab/l) when $x = a$. Similarly, if we consider the 1 ton rolling load between B and C at a distance of y from B, then

$$R_A = \frac{y}{l} \text{ and the bending moment at C } = \frac{y}{l}a.$$

This is a linear equation in y which varies from zero when $y = o$ up to a maximum value of (ab/l) when $y = b$.

Therefore the influence line for bending moment at the point C is a triangle having its maximum ordinate vertically below C of value (ab/l) as shown in Fig. 6 (b).

To Draw the Influence Line for Bending Moment at a Given Point on a Beam of Span l.

Draw the beam and underneath it draw a zero line for the influence line. Then if the point divides the span in the ratio $a : b$ as in Fig. 6 (a), erect an ordinate of magnitude (ab/l) vertically

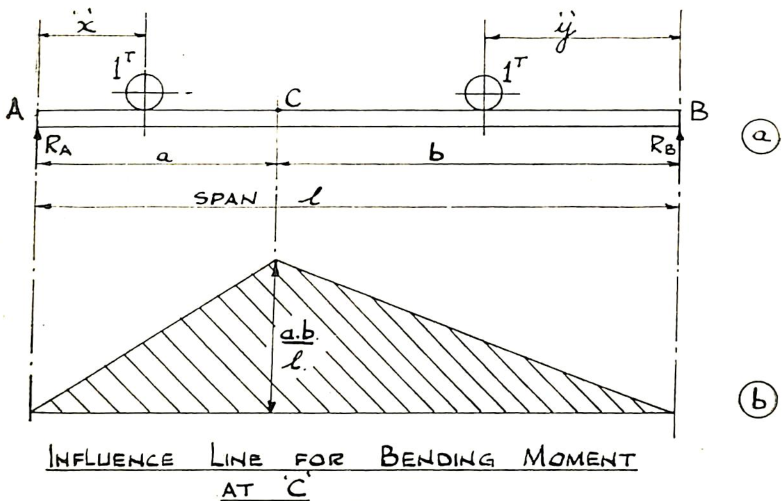
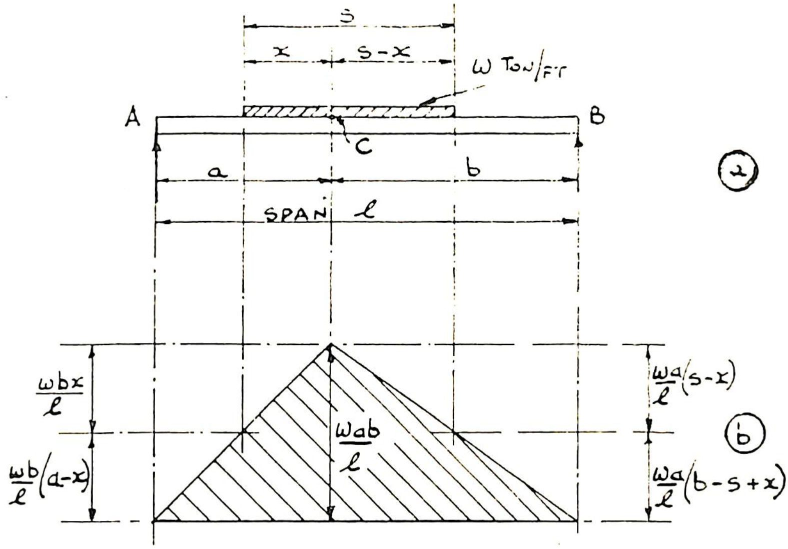


Fig. 6.



INFLUENCE LINE FOR BENDING MOMENT AT 'C'

Fig. 7.

below the point being considered. Join the tip of the ordinate to the zero line at each support, so that a triangle is formed with its maximum value vertically below the point being considered. This is the required influence line as in Fig. 6 (b).

Maximum Bending Moment due to an Equivalent Uniformly Distributed Live Load which is Shorter than the Span.

If the equivalent uniformly distributed live load is longer than the span of the beam, then the greatest bending moment at the point considered will occur when the load completely covers the span.

Now consider the beam AB of span l, in which the point under consideration C divides the span l in the ratio of $a : b$, when a short uniformly distributed live load of w tons per foot run and of total length s, crosses the span. In Fig. 7 (a) the load is shown with a length x on the part of the beam between A and C. It is required to find the value of x to give the maximum bending moment at C due to this load. The influence line for bending moment at C is drawn as shown in Fig. 7 (b).

Now the bending moment at C $= \dfrac{aws}{l}\left(b+x-\dfrac{s}{2}\right)-\dfrac{wx^2}{2}$

Differentiating this with respect to x, the value of x may be obtained which gives the maximum bending moment at C.

Differentiating we have $\dfrac{aws}{l} - \dfrac{2\,wx}{2} = 0 = as - xl.$

Therefore $\dfrac{x}{a} = \dfrac{s}{l}$. Similarly it may be shown that $\dfrac{s-x}{b} = \dfrac{s}{l}$

Therefore $\dfrac{x}{a} = \dfrac{s-x}{b}$. This shows that the point C divides the length of loading in the same ratio as it divides the span.

It will also be seen that in this case the ordinates to the influence line under the two ends of the load are equal.

$$i.e., \quad \frac{wb}{l}(a-x) = \frac{wa}{l}(b-s+x)$$

Therefore to find the maximum bending moment at a point due to a uniform load shorter than the span, divide the load by the point in the same ratio as the point divides the span.

To Find the Maximum Bending Moment which can occur on a Beam Subjected to a Given System of Wheel Loads.

The maximum bending moment will always occur directly under one of the loads, and the position of this load is required to give the maximum value of bending moment on the beam.

Fig. 8 shows a beam AB of span l for which the reactions are R_A and R_B as shown. A series of rolling loads are on the beam, the total loads to the left of the beam centre line being w and those to the right W. The maximum bending moment will occur under one of the loads nearer to the beam centreline. Assume that it is the wheel load s at a distance x from the beam centreline under which the maximum occurs. The centre of gravity of the whole wheel load system is $R = (w+W)$ at a distance c from the wheel s, and distances a and b from the points of application of w and W respectively.

Then we have $R_A = \dfrac{R}{l}\left(\dfrac{l}{2}+c-x\right)$ and

Bending moment at wheel $s = R_A\left(\dfrac{l}{2}+x\right) - w\,(a+c)$

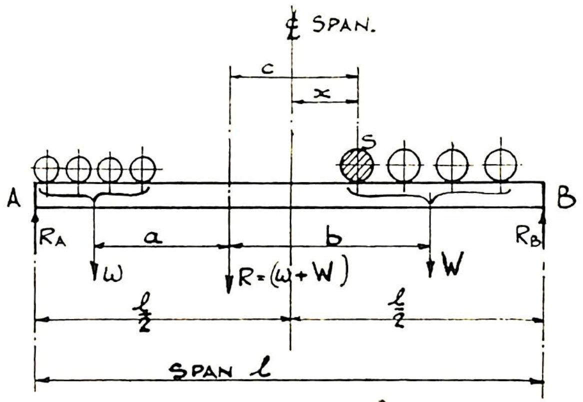


Fig. 8.

$$= \frac{R}{l}\left(\frac{l^2}{4} + \frac{cl}{2} + cx - x^2\right) - w\ (a+c)$$

For a maximum value, the differential coefficient of bending moment with respect to x must be zero.

Therefore R $(c-2x)/l = 0$ and $x = c/2$

Therefore the maximum bending moment occurs when the centre-line of the span bisects the distance between the wheel in question and the centre of gravity of the whole wheel system.

Example 4.—Find the maximum bending moment on a girder of 60 feet simply supported span when the wheel load system shown in Fig. 9 (a) rolls over it.

The distance of the centre of gravity of the whole wheel load system from the end 1 ton load is 21 feet, which is 4 feet from the innermost 2 ton load.

The wheel load system is shown on the beam in Fig. 9 (b) with the centreline of the span midway between the centre of gravity of the loads and the innermost 2 ton load.

The bending moment influence line is drawn for the point P (point of application of the 2 ton load) as shown in Fig. 9 (c).

Then the maximum bending moment on the girder occurs at the point P and is

$$(2 \times 14{\cdot}95)\left(\frac{28+23+18}{28}\right) + (14{\cdot}95)\left(\frac{22+17+12+7}{32}\right)$$

$$= \mathbf{100{\cdot}7\ tons\ feet.}$$

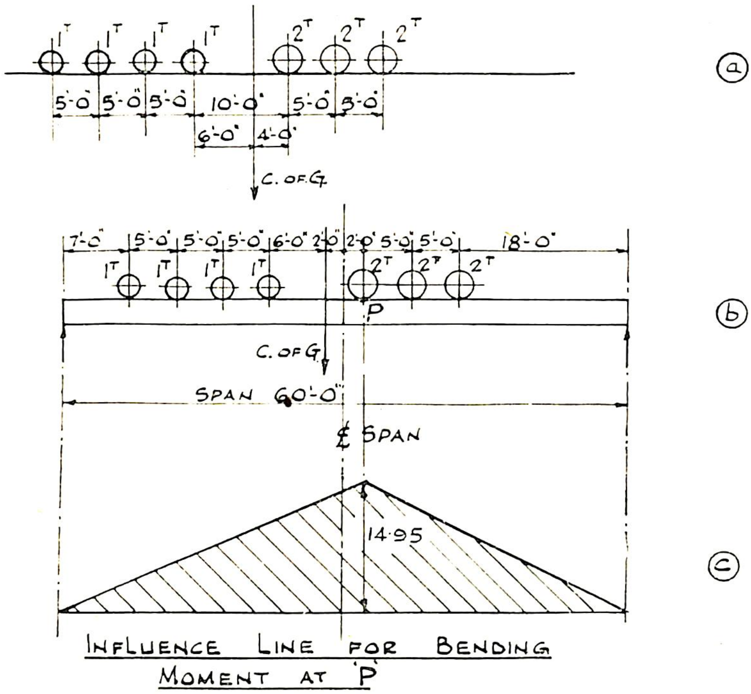


Fig. 9.

Example 5.—Find the value of the maximum bending moment at a point which is 20 feet from one end of a simply supported girder of 60 feet span. The girder is to carry equivalent live loads of $\frac{3}{4}$ ton per foot run longer than the span, or $1\frac{1}{2}$ tons per foot run 30 feet long, the two loads crossing the span at different times.

The girder is drawn in Fig. 10 (a) and the influence line for bending moment for the point under consideration P, is drawn in Fig. 10 (b).

When the equivalent live load of $\frac{3}{4}$ tons per foot run completely covers the span, the bending moment at point P $= \frac{1}{2} \times 60 \times \frac{40}{3} \times \frac{3}{4}$ $=$ **300 tons ft.** This is the greatest bending moment which can occur at point P, for any position of this load.

When the equivalent live load of $1\frac{1}{2}$ tons per foot run is carried on the span, the greatest value of the bending moment at point P occurs when the point P divides the load in the same ratio as it divides the span, *i.e.*, when the load is in the position shown in

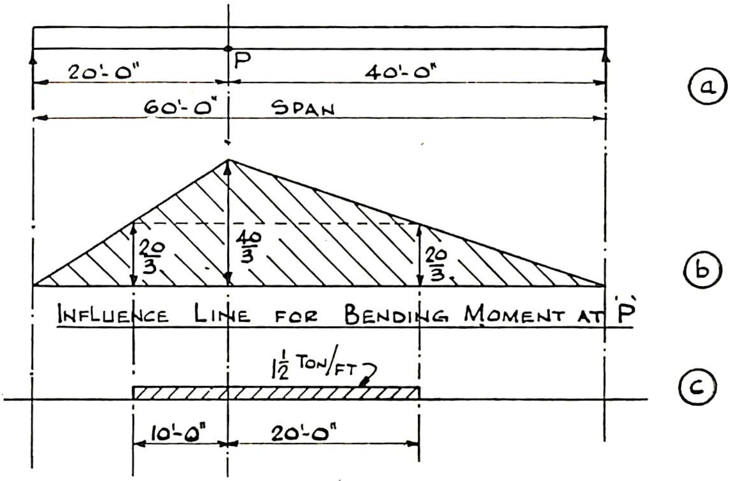


Fig. 10.

Fig. 10 (c). Therefore the greatest value of the bending moment at point P due to this load

$$= 1\tfrac{1}{2}\left\{\left(\frac{20}{3}\times 30\right) + \left(\tfrac{1}{2}\times\frac{20}{3}\times 30\right)\right\} = \textbf{450 tons ft.}$$

Therefore the maximum bending moment at the point P occurs under the shorter live load, and has a vaiue of 450 tons ft.

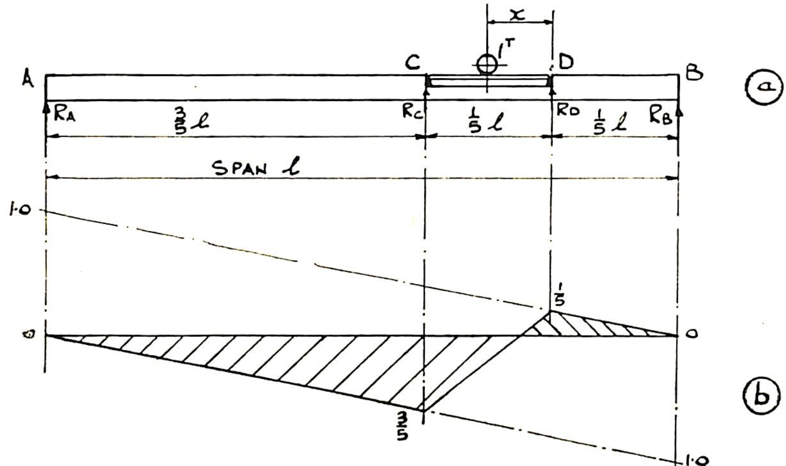
V.—INFLUENCE LINES FOR SHEAR IN BUILT-UP GIRDERS.

In built-up girders the load is not transmitted directly into a member, but is transmitted into the girder from cross-girders at the panel points. The influence lines for reaction and bending moment do not alter from those described previously, but the influence line for shear force is modified.

Fig. 11 (a) shows a girder AB of span l which is hollowed out for a length $l/5$ to carry a small independent girder CD.

From the notes on influence lines for shear force, it is seen that when the 1 ton rolling load is between A and C the shear influence line is the same as for the reaction R_B, and when the load is between D and B the influence line is the same as for reaction R_A.

Now consider the 1 ton rolling load on the small girder CD as in Fig. 11 (a).



INFLUENCE LINE FOR SHEAR IN THE GIRDER CD.

Fig. 11.

Then $R_A = \dfrac{(x + l/5)}{l}$ and $R_B = \dfrac{(4l/5 - x)}{l}$ and the small girder will cause reactions $R_C = 5x/l$ and $R_D = 1 - 5x/l$ on to the main girder.

Now the shear force at the load is $R_A - R_C = \dfrac{(x + l/5)}{l} - 5x/l$

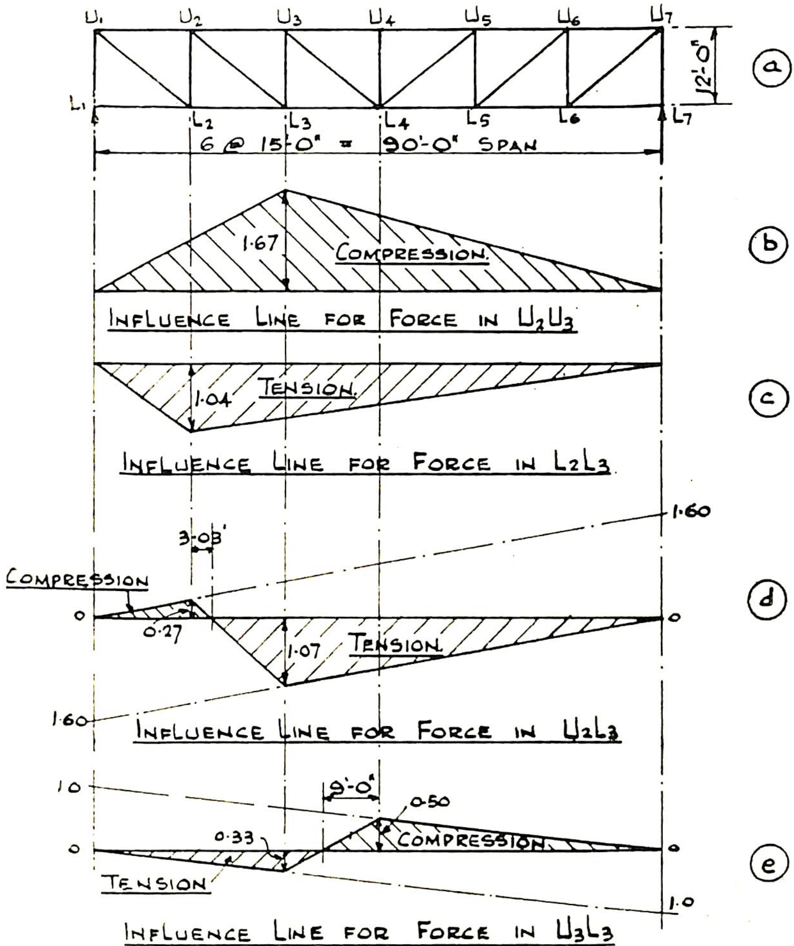
$$= \frac{(l/5 - 4x)}{l}$$ which is a linear equation in x.

Therefore the influence line for shear for the girder CD is as shown in Fig. 11 (b) since it must be a straight line between C and D.

Also it will be seen from this that the shear force influence line for a member in a braced girder must be a straight line between the panel points where the loads are transmitted into the girder.

Influence Lines for an "N" Girder.

Example 6.—Draw the influence lines for the forces in the members U_2U_3, L_2L_3, U_2L_3 and U_3L_3 of the frame shown in Fig. 12 (a). The loads are carried by cross girders at the lower panel points.



U1
U2
U3
U4
U5
U6
U7
12'-0"
(a)
L1
L2
L3
L4
L5
L6
L7
6 @ 15'-0" = 90'-0" SPAN
1·67
COMPRESSION.
(b)
INFLUENCE LINE FOR FORCE IN U2U3
1·04
TENSION.
(c)
INFLUENCE LINE FOR FORCE IN L2L3
1·60
3·03'
COMPRESSION
0
0·27
1·07
TENSION.
0
(d)
1·60
INFLUENCE LINE FOR FORCE IN U2L3
1·0
9'-0"
0·50
0·33
COMPRESSION.
0
0
(e)
TENSION
1·0
INFLUENCE LINE FOR FORCE IN U3L3

Fig. 12.

Force in U_2U_3.

Force in the member is the bending moment at point L_3 divided by the depth of the girder, and therefore the influence line for the force is the bending moment influence line for point L_3 divided by the depth of the girder.

Therefore the influence line encloses a triangle of which the maximum value is $\left(\frac{30 \times 60}{90 \times 12}\right) =$ **1·67 tons,** as shown in Fig. 12 (b). The member U_2U_3 is always in compression.

Force in L_2L_3.

Similarly to the above member, the influence line for the force in L_2L_3 is the bending moment influence line for the point U_2 divided by the depth of the girder. The influence line encloses a triangle of which the maximum value is $\left(\frac{15 \times 75}{90 \times 12}\right) =$ **1·04 tons** as shown in Fig. 12 (c). The member L_2L_3 is always in tension.

Force in U_2L_3.

This member carries the shear in the second panel, and the force in it is the vertical shear in this panel $\times \left(\frac{\text{length of } U_2L_3}{\text{depth of girder}}\right)$

$$= 1{\cdot}60 \times \text{vertical shear.}$$

Therefore the influence line for the force in U_2L_3 is $1{\cdot}6 \times$ (the shear force influence line for the second panel). This is drawn as described previously but using **vertical ordinates of 1·60 instead of 1·0 tons,** as shown in Fig. 12 (d).

When the load is to the left of point L_2, the member U_2L_3 is in compression, and when the load is to the right of L_3, the member is in tension.

Force in U_3L_3.

By resolving forces at the joint U_3 we find that the force in member U_3L_3 is the vertical component of the force in member U_3L_4, and is, therefore, the vertical shear in the third panel. Therefore, the influence line for the force in member U_3L_3 is the shear force influence line for the third panel which is shown in Fig. 12 (e).

When the load is to the left of point U_3, the member U_3L_3 is in tension, and when the load is to the right of U_4, the member is in compression.

Example 7.—Draw the influence lines for the forces in the members U_2U_3, L_2L_3, U_2L_3, U_3L_3 and U_4L_4 for the frame shown in Fig. 13 (a). The loads are carried by cross-girders at the upper panel points.

The influence lines for the forces in the members U_2U_3, L_2L_3 and U_2L_3 are exactly the same as for the previous example, as shown in Figs. 13 (b), (c) and (d).

Force in U_3L_3.

By resolving forces at the point L_3 we find that the force in member U_3L_3 is the vertical component of the force in member U_2L_3, and is, therefore, the vertical shear in the second panel. Therefore the influence line for the force in member U_3L_3 is the shear force influence line for the second panel which is shown in Fig. 13 (e). When the load is to the left of point U_2 the member U_3L_3 is in tension, and when the load is to the right of point U_3 the member is in compression.

Force in U_4L_4.

When the load is between U_1 and U_3, and between U_5 and U_7, there can be no force in the member U_4L_4. Also, when the load is directly above U_4, the force in the member will be 1 ton compression if the load is 1 ton. Also, as the load moves from U_3 to U_4 (or from U_5 to U_4) the force in member U_4L_4 increases linearly, in exactly the same way as the influence line for reaction.

Therefore the influence line for the force in the member U_4L_4 encloses a triangle of which the maximum ordinate is **1·0 tons,** as shown in Fig. 13 (f).

Influence Lines for a Warren Girder.

Example 8.—Draw the influence lines for the forces in the members U_2U_3, L_2L_3 and U_3L_2 of the girder shown in Fig. 14 (a) in which all the members are of the same length. The loads are transmitted into the Warren girder from cross girders at the upper panel points.

Force in U_2U_3.

Similarly to the previous examples we see that the influence line for the force in the member U_2U_3 is the bending moment influence line for the point L_2, divided by the depth of the girder. This influence line encloses a triangle of which the maximum ordinate of 1·21 tons is vertically below point L_2. But L_2 lies between the two upper panel points U_2 and U_3, and we have already

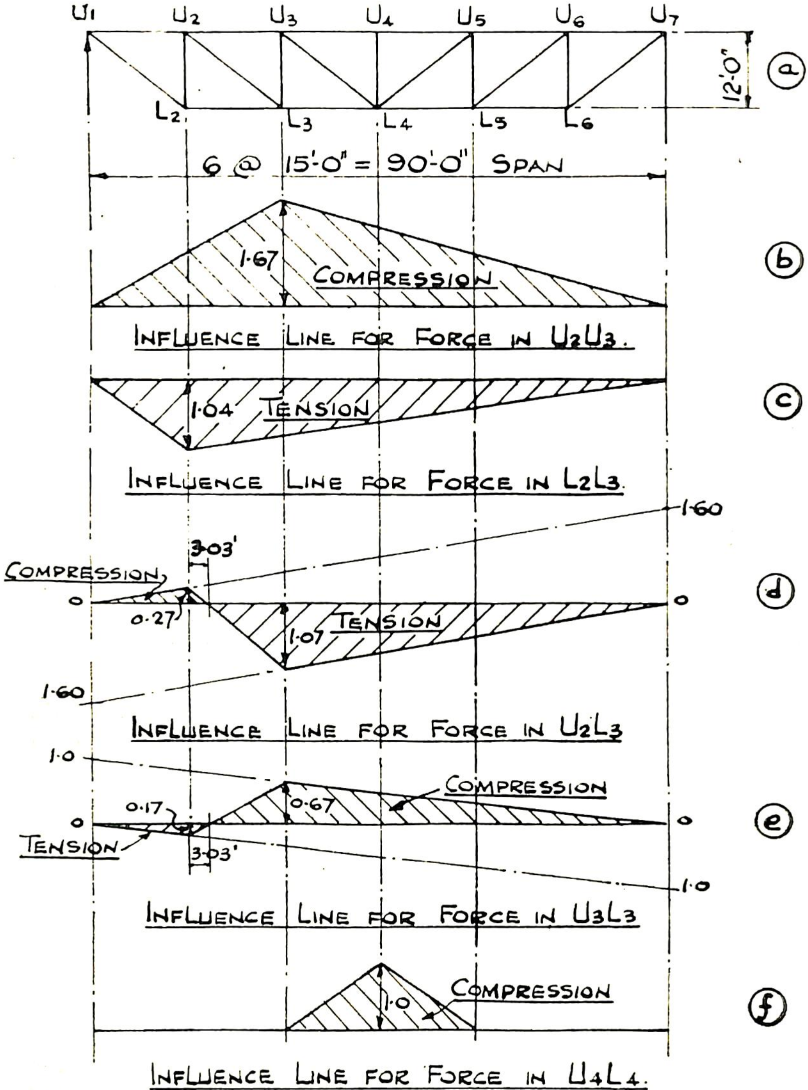
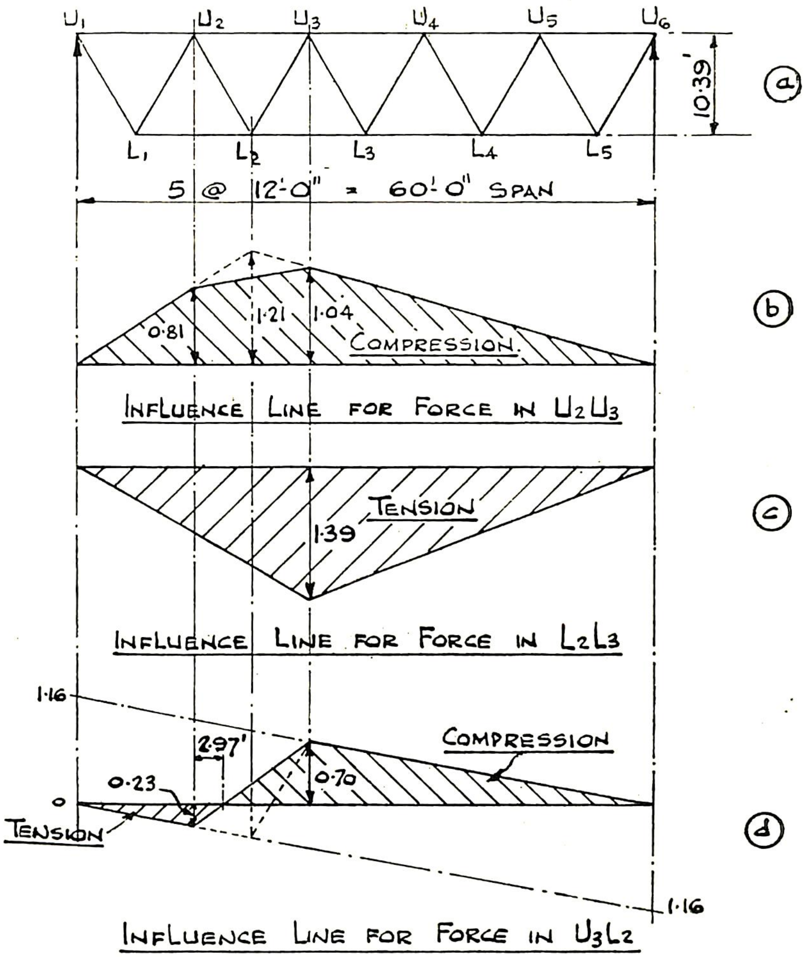

U1
U2
U3
U4
U5
U6
U7
L2
L3
L4
L5
L6
12'-0"
(a)
6 @ 15'-0" = 90'-0" SPAN
1·67
COMPRESSION
(b)
INFLUENCE LINE FOR FORCE IN U2U3.
1·04
TENSION
(c)
INFLUENCE LINE FOR FORCE IN L2L3.
1·60
3·03'
COMPRESSION
0
0·27
1·07
TENSION
0
(d)
1·60
INFLUENCE LINE FOR FORCE IN U2L3
1·0
COMPRESSION
0·17
0·67
0
0
TENSION
3·03'
(e)
1·0
INFLUENCE LINE FOR FORCE IN U3L3
1·0
COMPRESSION
(f)
INFLUENCE LINE FOR FORCE IN U4L4.

Fig. 13.



U1
U2
U3
U4
U5
U6
10·39'
a
L1
L2
L3
L4
L5
5 @ 12'-0" = 60'-0" SPAN
1·21
1·04
0·81
COMPRESSION.
b
INFLUENCE LINE FOR FORCE IN U2U3
TENSION
1·39
c
INFLUENCE LINE FOR FORCE IN L2L3
1·16
2·97'
COMPRESSION
0·23
0·70
0
TENSION
d
1·16
INFLUENCE LINE FOR FORCE IN U3L2

Fig. 14.

seen that the influence line between any two adjacent cross girders must be a straight line. Therefore the peak of the triangular influence line is cut off where the straight line joins the influence line vertically below the points U_2 and U_3. The final shape of the influence line is shown in Fig. 14 (b) where the maximum ordinate is **1.04 tons** vertically below U_3. The member U_2U_3 is always in compression.

Force in L_2L_3.

Again, from the previous examples, the influence line for the force in member L_2L_3 is the bending moment influence line for the point U_3 divided by the depth of the girder. This influence line encloses a triangle of which the maximum ordinate of **1·39 tons** lies vertically below the point U_3, as shown in Fig. 14 (c). The member L_2L_3 is always in tension.

Force in U_3L_2.

The member U_3L_2 carries the shear force in its own panel, and the influence line for the member appears to be the influence line

for vertical shear force in the panel $U_3L_2 \times \left(\dfrac{\text{length of member}}{\text{depth of girder}}\right)$

$= 1{\cdot}16 \times$ shear force influence line for the panel U_3L_2. However, if this influence line is drawn, it is not a straight line below the panel points U_2 and U_3, and therefore the influence line must be modified as shown in Fig. 14 (d). When the load is to the left of U_2 the member U_3L_2 is in tension, and when the load is to the right of U_3, the member is in compression.

Influence Lines for a "K" Girder.

Example 9.—Draw the influence lines for the forces in the members L_0L_1, L_1L_2, U_1U_2, L_0U_1, M_1L_1, M_1U_2, M_1L_2, M_2U_2, M_2L_2 and U_3L_3 of the "K" girder shown in Fig. 15 (a), if the loads are carried by cross-girders at the lower panel points.

The influence lines for the forces in the members L_0L_1, L_1L_2 and U_1U_2 may be obtained as described above and they are shown in Figs. 15 (b) and (c).

Force in L_0U_1.

It may be seen that the force in the member L_0U_1 is the bending moment at the point L_1 divided by 15·35 feet, and therefore the required influence line for the force in the member is the bending

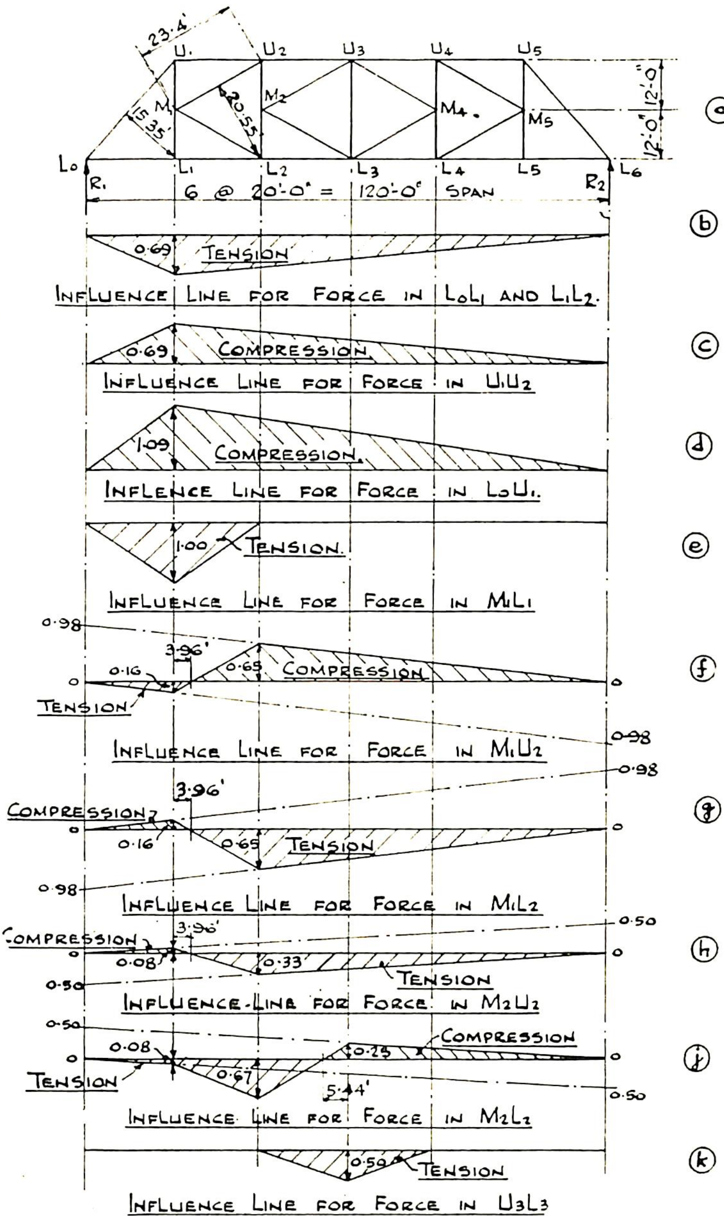

23.4'
U1
U2
U3
U4
U5
M1
M2
M4
M5
20.55
15.35'
12'-0"
12'-0"
L0
L1
L2
L3
L4
L5
L6
R1
R2
6 @ 20'-0" = 120'-0" SPAN
(a)
(b)
0.69
TENSION
INFLUENCE LINE FOR FORCE IN L0L1 AND L1L2.
(c)
0.69
COMPRESSION
INFLUENCE LINE FOR FORCE IN U1U2
(d)
1.09
COMPRESSION
INFLENCE LINE FOR FORCE IN L0U1.
(e)
1.00
TENSION
INFLUENCE LINE FOR FORCE IN M1L1
(f)
0.98
3.96'
0.16
0.65
COMPRESSION
TENSION
0
0.98
INFLUENCE LINE FOR FORCE IN M1U2
(g)
0.98
3.96'
COMPRESSION
0.16
0.65
TENSION
0.98
INFLUENCE LINE FOR FORCE IN M1L2
(h)
0.50
3.96'
COMPRESSION
0.08
0.33
TENSION
0.50
INFLUENCE-LINE FOR FORCE IN M2U2
(j)
0.50
0.08
0.25
COMPRESSION
TENSION
0.67
5.44'
0.50
INFLUENCE LINE FOR FORCE IN M2L2
(k)
0.50
TENSION
INFLUENCE LINE FOR FORCE IN U3L3

Fig. 15.

moment influence line for the point L_1 divided by 15·35 feet. Therefore the influence line encloses a triangle of which the maximum ordinate lies vertically below L_1 of value **1·09 tons.** This member is always in compression the influence line being shown in Fig. 15 (d).

Force in M_1L_1.

When the 1 ton rolling load is at L_0, and when it is either at, or to the right of L_2, there is no force at all in the member M_1L_1. When the load is at L_1, the force in the member is 1 ton tension. As the load travels from L_0 to L_1 and from L_2 to L_1, the force in the member M_1L_1 increases linearly in exactly the same way as if L_0L_1 and L_2L_1 were both simply supported spans of an ordinary beam and L_1 was a support. Therefore the influence line for the force in the member M_1L_1 is shown in Fig. 15 (e), the member being in tension when loaded.

Force in M_1U_2.

By taking moments about the point L_2 we have

$$\text{Force in } M_1U_2 = \frac{40\,R_1 - 24\,F}{20{\cdot}55}$$

where F is the force in member U_1U_2 when the load is to the right of L_2.

Now consider the unit load at a distance of x ft. from the right-hand reaction R_2.

$$\text{Then } R_1 = \frac{x}{120} \text{ tons and } F = \frac{x}{120} \times \frac{20}{24} = \frac{x}{144} \text{ tons.}$$

$$\therefore \text{ Force in } M_1U_2 = \frac{\dfrac{40x}{120} - \dfrac{24x}{144}}{20{\cdot}55} = \frac{x}{123{\cdot}3} \text{ tons compression.}$$

This is a linear relation but is only true as long as x is at on to the right of L_2, with a maximum value of **0·65 tons** compression at L_2.

Similarly, if we consider the load to the left of point L_1, we obtain a straight line for the influence line between L_0 and L_1 with a maximum value of **0·16 tons** tension at L_1. It has already been shown that between any two cross-girders the influence line must be a straight line, and therefore the influence line will be as shown in Fig. 15 (f).

Force in M_1L_2.

By taking moments about the point U_2, and then continuing in a similar manner to that adopted for member M_1U_2 above, an influence line for the force in member M_1L_2 is obtained as shown in Fig. 15 (g). It will be noted that the magnitudes of the ordinates for this influence line are exactly the same as those for the member M_1U_2, but that the ordinates are of opposite sign as would be expected.

Force in M_2U_2.

Since the loads are carried by cross-girders at the lower panel points, the force in the member M_2U_2 is always the force in member

$$M_1U_2 \text{ multiplied by } \left(\frac{\text{length of member } M_2U_2}{\text{length of member } M_1U_2}\right)$$
$$= 0{\cdot}513 \times \text{force in member } M_1U_2.$$

Therefore the influence line for the force in member M_2U_2 is the influence line for the force in member M_1U_2 multiplied by 0·513, as shown in Fig. 15 (h). When the load is to the right of L_2 the member M_2U_2 is in tension, and when the load is to the left of L_1 the member is in compression.

It will be noted that if the side lines of the Fig. 15 (h) are extended to cut the vertical lines through the supports, the ordinates at these points are 0·50 tons which is correct, since the member M_2U_2 carries one-half of the shear force of the second panel of the girder.

Force in M_2L_2.

When the 1 ton rolling load is between L_0 and L_1 and between L_3 and L_6, the member M_2L_2 carries one-half of the shear force of the second panel of the girder. This may be shown in a similar manner to that adopted for the member M_2U_2. When the load is between L_0 and L_1, the member M_2L_2 is in tension, and when the load is between L_3 and L_6 the member is in compression.

Now consider the 1 ton load between the panel points L_1 and L_2, at a distance of x from L_2. Then by solving the girder, we find that there is a tensile force of $\left(\frac{2}{3} - \frac{7x}{240}\right)$ tons in member M_2L_2. This is a linear relation in x.

For the point L_2, substitute $x = 0$ to obtain the force in the member M_2L_2 of 0·67 tons tension when the load is at L_2. Therefore 0·67 tons is the ordinate of the influence line vertically below L_2.

For the point L_1 substitute $x = 20$ feet to obtain the force in the member M_2L_2 of 0·08 tons tension, which agrees with the height of the influence line vertically below L_1 which is already known.

Now consider the 1 ton load between the panel points L_2 and L_3 at a distance of x from L_2. Again solving the girder we obtain a tensile force of $\left(\frac{2}{3} - \frac{11x}{240}\right)$ tons in the member M_2L_2, which is another linear relation in x.

Substituting $x = 0$ we obtain a force in the member M_2L_2 of 0·67 tons tension which agrees with the height of the influence line already obtained below L_2, and substituting $x = 20$ feet, the height of the influence line vertically below L_3 is obtained as 0·25 tons compression. This agrees with the part of the influence line already known, when the member carries half of the shear force of the second panel of the girder. Therefore the influence line for the member M_2L_2 is as shown in Fig. 15 (j).

Force in U_3L_3.

When the 1 ton rolling load is between L_0 and L_2, and between L_4 and L_6, there is no force at all in the member U_3L_3.

Now place the 1 ton load between L_2 and L_3 at a distance of x from L_3. Then the force in U_3L_3 is found to be $\left(\frac{1}{2} - \frac{x}{40}\right)$ tons tension. For the point L_3, $x = 0$ and the force in member U_3L_3 is 0·50 tons tension. For the point L_2, since $x = 20$ feet, there is no force in the member as already decided, and since the above equation is linear in x, the influence line is a straight line as shown in Fig. 15 (k). It will be seen that if the load is between L_3 and L_4 the equation will be the same as that above. The influence line therefore encloses a triangle.

VI.—INFLUENCE LINES FOR BUILT-UP GIRDERS WITH CURVED BOOMS.

In the girder with the curved top boom shown in Fig. 16 (a), the influence lines for the forces in the members U_1L_2 and U_2L_2 do not depend entirely on the shear force in the second panel of the girder, as in the case of previous examples. The point P which is at a distance b from the left-hand reaction R_A is the point of intersection of the two members U_1U_2 and L_1L_2. The distances

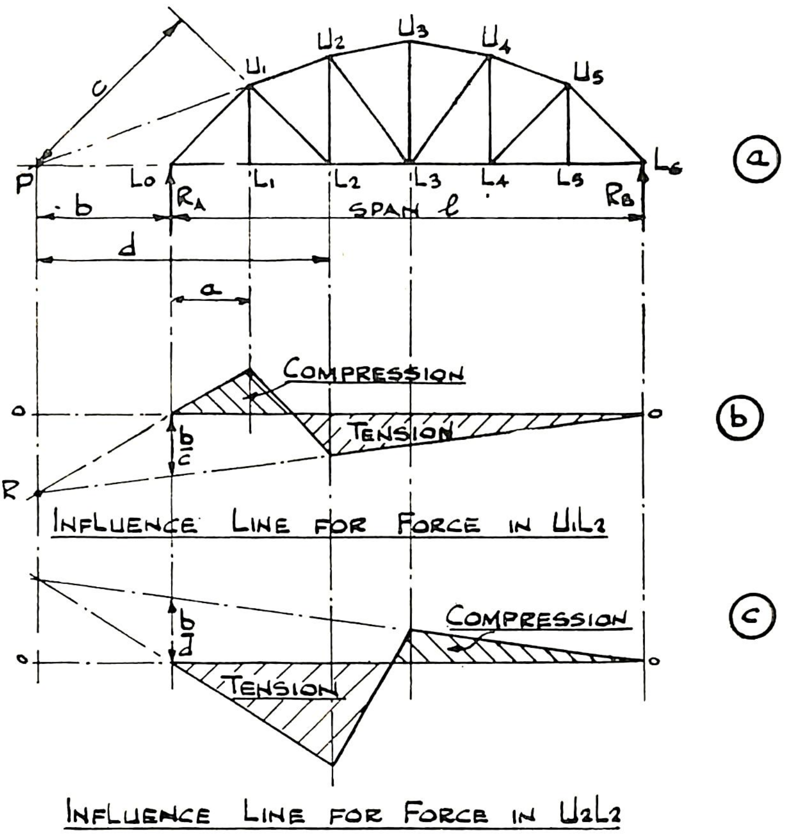


Fig. 16.

c and d are the perpendicular distances from the point P to the members U_1L_2 and U_2L_2 respectively.

Force in U_1L_2.

If we take a section through the girder, to cut the member U_1L_2 and any other two members (say L_1L_2 and U_1U_2) then, for the equilibrium of the girder at this section, it will be seen that the force in member U_1L_2 is the bending moment at the point P divided by the distance c.

Now let the 1 ton rolling load lie between the points L_2 and L_6 at a distance of x from L_6. Then the reaction $R_A = x/l$ and considering the equilibrium of the girder to the left of the section cutting the girder, the bending moment at point P $= x.b/l$, and the force in the member U_1L_2 is $x.b/l.c$ tension.

This is a linear equation in x and, as we have seen from previous work, represents the influence line for the force in the member U_1L_2 when the load is between the points L_2 and L_6.

Now consider the 1 ton rolling load between L_0 and L_1 at a distance of x from L_0. Then the reaction $R_B = x/l$ and the bending moment at the point P (considering the equilibrium of the girder to the right of the section cutting the girder) is $x(l+b)/l$. and the force in the member U_1L_2 is $\frac{x}{l}\left(\frac{l+b}{c}\right)$ compression. This represents a linear equation in x and is the equation to the influence line for the force in the member U_1L_2 when the load is to the left of point L_1.

When the rolling load is in the panel L_1L_2, we have already seen that the influence line is a straight line. Therefore if actual values are substituted for the symbols b, c, l, etc., the influence line may be drawn as shown in Fig. 16 (b). However, a graphical solution can be evolved for this influence line which eliminates a lot of the above working.

Graphical Construction for the Influence Line for the Force in the Member U_1L_2.

If the function $\frac{x}{l} \cdot \frac{b}{c}$ be continued up to the point where $x = (l+b)$, the ordinate at this point becomes $\frac{b\,(l+b)}{lc}$ vertically below P, and it indicates tension since the function indicates tension.

Now if we put $x = -b$ in the function $\frac{x}{l}\left(\frac{l+b}{c}\right)$ where the function represents compression we find the height of the ordinate vertically below P as $= \frac{-b\,(l+b)}{lc}$ the negative sign indicating tension.

Therefore we know that the two separate parts of the final influence line both meet at a point vertically below the point P, at a distance of $\frac{b\,(l+b)}{lc}$ on the tension side of the zero line.

If this point is joined to the zero line under reaction R_B, it is in the same line as the part of the influence line between L_2 and L_6, and the ordinate where this line cuts the vertical line through R_A is b/c as shown in Fig. 16 (b).

Therefore to draw the influence line for the force in member U_1L_2, erect an ordinate of value (b/c) tension vertically below the reaction which is nearer to the point of intersection (P) of the boom members. Join this point to the zero line under the other reaction and continue the line to cut the vertical through the point of intersection in a point {R on Fig. 16 (b) *}. From this point (R) draw a line through the zero point vertically below the nearer reaction and continue until it cuts the vertical through L_1. Then complete the influence line as described above and as shown in Fig.* 16 (b).

Force in U_2L_2.

By adopting a similar argument to that used for the member U_1L_2, we find that the influence line for the force in the member U_2L_2 is a straight line when the rolling load is between L_3 and L_6, in which case the force in the member U_2L_2 is compressive. It is also a straight line when the rolling load is between L_0 and L_2, when the member is in tension, and also a straight line between L_2 and L_3. A graphical construction may again be evolved which is very similar to that described above.

In this case an ordinate of value b/d is erected on the *compression* side of the zero line vertically below the reaction nearer to the point P. The construction is then the same as for the previous member, except that the line joining the tension and compressive sides of the influence line is now in the panel L_2L_3 and not in the panel L_1L_2 as it was for the previous member. The reason for this has already been seen in "Influence Lines for Shear in Built-up Girders."

The final influence line for the member U_2L_2 is shown in Fig. 16 (c).

Note.—If the reader finds difficulty in deciding whether the ordinate to be erected is tensile or compressive, it is easier to draw the influence line first and then to apply a 1 ton load at one point in the girder. This gives the sign of the force in the member for a load at this point, and so the tensile and compressive parts of the influence line may be decided.

Influence Lines for Pratt Girder with Curved Top Boom.

Example 10.—Draw the influence lines for the forces in the members U_1L_1, U_1U_2, L_1L_2, U_1L_2 and U_2L_2 of the girder shown in

Fig. 17 (a). The loads are carried by cross-girders at the lower panel points.

Force in U_1L_1.

When the 1 ton rolling load is between L_2 and L_6 there can be no force at all in the member U_1L_1, and also when the load is at the reaction R_A there is no force in the member. When the load is at the point L_1 there is a tensile force in the member of 1·0 tons, and since it has already been shown for a case of this kind, that the influence line between any two cross-girders must be a straight line, then the influence line must be as shown in Fig. 17 (b).

Force in U_1U_2.

The force in member U_1U_2 is the bending moment at the point L_2 divided by the perpendicular distance from L_2 to the member U_1U_2 which is 18·9 feet. Therefore the influence line for the force in member U_1U_2 is the bending moment influence line for the point L_2 divided by 18·9 feet. Therefore the influence line encloses a triangle with a maximum ordinate of value **1·06 tons** vertically below L_2 as shown in Fig. 17 (c). This member is always in compression.

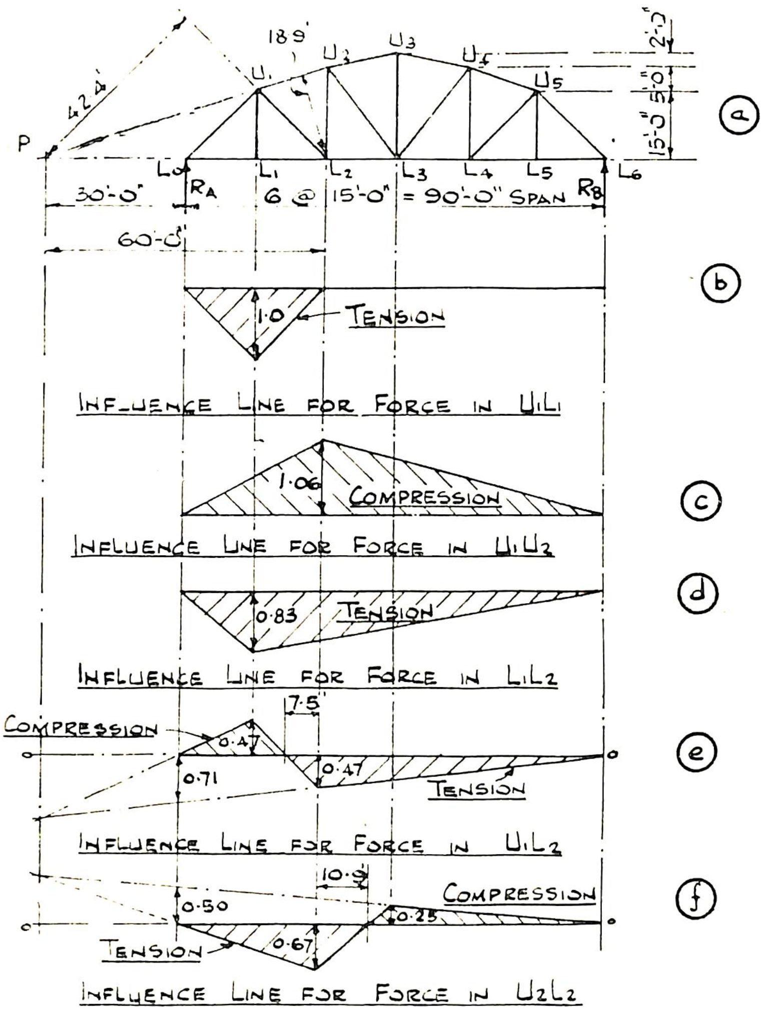
Force in L_1L_2.

The force in member L_1L_2 is the bending moment at the point U_1 divided by the length of member U_1L_1, and therefore the influence line for the force in member L_1L_2 is the bending moment influence line for the point U_1 divided by 15 feet. Therefore the influence line encloses a triangle with a maximum ordinate of value **0·83 tons** as shown in Fig. 17 (d). The member L_1L_2 is always in tension.

Force in U_1L_2.

The influence line required is easily obtained by the graphical construction which is explained above. The boom members U_1U_2 and L_1L_2 meet at a point P distance 30 feet from the reaction R_A, and the perpendicular distance from the point P to the member U_1L_2 is 42·4 feet, as shown in Fig. 17 (a).

The ordinate of value $\dfrac{30}{42{\cdot}4} = 0{\cdot}71$ tons has been erected on the tension side of the zero line, vertically below R_A in Fig. 17 (e). A line is drawn from the zero point vertically below R_B through the tip of the ordinate below R_A and continued to cut the vertical line through P. From this point a line is drawn through the zero



18.9'
42.4'
P
U1
U2
U3
U4
U5
L0
L1
L2
L3
L4
L5
L6
RA
RB
30'-0"
60'-0"
6 @ 15'-0" = 90'-0" SPAN
2'-0"
5'-0"
15'-0"
(a)
(b)
1.0
TENSION
INFLUENCE LINE FOR FORCE IN U1L1
(c)
1.06
COMPRESSION
INFLUENCE LINE FOR FORCE IN U1U2
(d)
0.83
TENSION
INFLUENCE LINE FOR FORCE IN L1L2
(e)
7.5'
COMPRESSION
0.47
0.71
0.47
TENSION
INFLUENCE LINE FOR FORCE IN U1L2
(f)
10.9'
COMPRESSION
0.50
0.25
0.67
TENSION
INFLUENCE LINE FOR FORCE IN U2L2

Fig. 17.

below R_A and continued to cut the vertical line through L_1. The influence line is joined by the diagonal between L_1 and L_2, and the final diagram is as shown in Fig. 17 (e).

Force in U_2L_2.

Again using the graphical construction, erect an ordinate of value $\frac{30}{60}$ = 0·50 tons on the compression side of the zero line vertically below R_A. Then complete the diagram in a similar manner to that described for the member U_1L_2 except that the diagonal joining the two sides of the influence line now lies between the points L_2 and L_3. The final influence line for the force in the member U_2L_2 is shown in Fig. 17 (f).

Example 11.—Draw the influence lines for the forces in the members U_1L_0, U_1U_2, U_2U_3, L_0L_1, L_1L_2, U_2L_2, L_1U_2 and U_1L_1 of the girder shown in Fig. 18 (a). The loads are carried by cross-girders at the lower panel points.

Force in U_1L_0.

When the 1 ton rolling load lies between L_1 and L_4, the force in member U_1L_0 is the reaction at L_0 multiplied by

$$\left(\frac{\text{length of member } U_1L_0}{\text{height of } U_1 \text{ above } L_0L_1}\right) = 1{\cdot}40 \times \text{reaction at } L_0.$$

This part of the influence line, therefore, will be linear, rising from zero below the reaction at L_4 up to a maximum value of **1·10 tons** vertically below L_1. It has also been shown previously that between the cross-girders L_0 and L_1 the influence line must be linear. Therefore the final shape of the influence line will be as shown in Fig. 18 (b). The member U_1L_0 is always in compression.

Force in U_1U_2.

The force in member U_1U_2 is the bending moment at the point L_1 divided by the perpendicular distance from the point L_1 to the member U_1U_2. Therefore the influence line for the force in member U_1U_2 is the influence line for bending moment at the point L_1 divided by 8·48 feet. This encloses a triangle with a maximum ordinate of **1·39 tons** vertically below the point L_1, as shown in Fig. 18 (c). This member is always in compression.

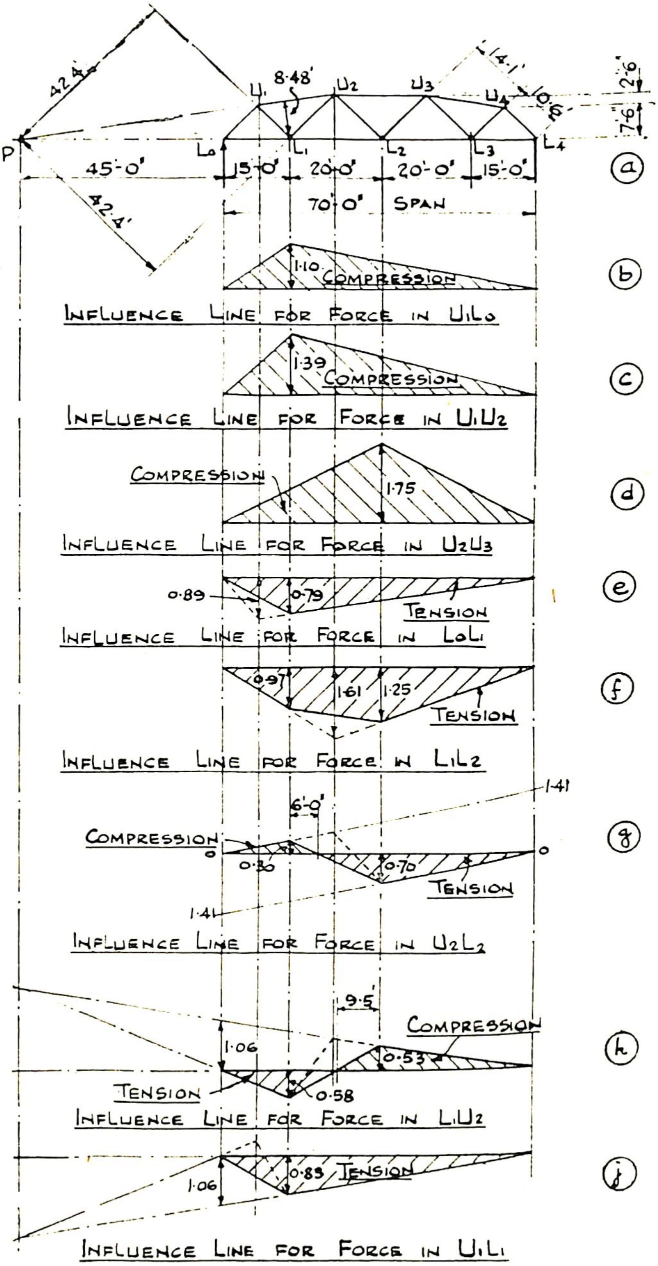

42.4'
8.48'
U_1
U_2
U_3
U_4
14.1'
10.6'
2'6"
7'6"
P
L_0
L_1
L_2
L_3
L_4
45'-0"
15'-0"
20'-0"
20'-0"
15'-0"
42.4'
70'-0" SPAN
(a)
1.10
COMPRESSION
(b)
INFLUENCE LINE FOR FORCE IN U_1L_0
1.39
COMPRESSION
(c)
INFLUENCE LINE FOR FORCE IN U_1U_2
COMPRESSION
1.75
(d)
INFLUENCE LINE FOR FORCE IN U_2U_3
0.89
0.79
TENSION
(e)
INFLUENCE LINE FOR FORCE IN L_0L_1
0.97
1.61
1.25
TENSION
(f)
INFLUENCE LINE FOR FORCE IN L_1L_2
1.41
6'-0"
COMPRESSION
0
0.30
0.70
0
TENSION
1.41
(g)
INFLUENCE LINE FOR FORCE IN U_2L_2
9.5'
COMPRESSION
1.06
0.53
TENSION
0.58
(h)
INFLUENCE LINE FOR FORCE IN L_1U_2
1.06
0.83
TENSION
(j)
INFLUENCE LINE FOR FORCE IN U_1L_1

Fig. 18.

Force in U_2U_3.

The force in this member is the bending moment at the point L_2 divided by the depth of the girder at midspan. Therefore the required influence line is the influence line for bending moment at the point L_2 divided by 10 feet. The final influence line encloses a triangle with a maximum ordinate of **1·75 tons** at the centre of the girder as shown in Figs. 18 (d). The member U_2U_3 is always in compression.

Force in L_0L_1.

This force is the bending moment at point U_1 divided by the height of U_1 above the member L_0L_1. Therefore the influence line for the force in member L_0L_1 is the bending moment influence line for the point U_1 divided by 7·5 feet. This would give a triangular influence line with the maximum ordinate of value **0·89 tons** vertically below U_1. However, it has already been seen that between any two cross-girders (*i.e.*, points to L_0 and L_1) the influence line must be linear, and so the required influence line is as drawn in Fig. 18 (e). The member L_0L_1 is always in tension.

Force in L_1L_2.

Since the force in L_1L_2 is the bending moment at the point U_2 divided by the height of U_2 above the member L_1L_2, then the required influence line is the bending moment influence line for the point U_2 divided by 10 feet. The influence line would then enclose a triangle with its maximum ordinate of **1·61 tons** vertically below U_2, but in this case the influence line must be linear between the cross-girders at points L_1 and L_2. Therefore the final shape of the influence line will be as shown in Fig. 18 (f). This member L_1L_2 is always in tension.

Force in U_2L_2.

The influence line for this member *appears* to be the influence line for vertical shear force between the points U_2 and L_2, multiplied

$$\text{by } \left(\frac{\text{length of member } U_2L_2}{\text{height of } U_2 \text{ above } L_1L_2}\right).$$

The heights of the ordinates to be erected will therefore be **1·41 tons** vertically below each reaction, and the two triangles may be completed, one on each side of the zero line as previously described. It would seem that the line joining the two sides of the influence line should go between U_2 and L_2 as shown dotted in Fig. 18 (g), but it must cross over between the points L_1 and L_2

so that the influence line between these two cross-girders is linear. When the load is above the left-hand part of the influence line, the member U_2L_2 is in compression, and when the load is above the right-hand part, the member is in tension, as indicated in Fig. 18 (g).

Force in L_1U_2.

Since the two boom members U_1U_2 and L_1L_2 meet at the point P, as shown in Fig. 18 (a), then the force in member L_1U_2 will be the bending moment at the point P divided by the distance from point P to the member L_1U_2. It will be seen that for this member, a graphical construction may be used which is identical with those used in the previous example. This is drawn in Fig. 18 (h) as described below.

An ordinate of $\left(\frac{45 \cdot 0}{42 \cdot 4}\right)$ tons $=$ **1·06 tons** is erected on the compression side of the zero line, and then a line is drawn from the other zero point through the ordinate and continued until it cuts the vertical line through P. This point is then joined to the nearer zero at a reaction and continued until it cuts the vertical line through L_1. Now the line joining the two halves of the influence line would appear to go between the points L_1 and U_2, but, so that the influence line is linear between the cross-girders the joining line must go between L_1 and L_2. The final influence line is shown in Fig. 18 (h).

Force in U_1L_1.

Since the two boom members again intersect at the point P, the graphical construction may be adopted to find the influence line for the force in member U_1L_1. Therefore erect an ordinate of value $\left(\frac{45 \cdot 0}{42 \cdot 4}\right)$ $=$ **1·06 tons** on the tension side of the zero line and vertically below the reaction at L_0. Join this point to the opposite zero. The part of this line between L_1 and L_4 will be part of the required influence line, and since the influence line between L_0 and L_1 must be linear, then the final shape of the influence line is known without continuing the graphical construction. The required influence line is shown in Fig. 18 (j), the member U_1L_1 always being in tension.

7.—INFLUENCE LINES FOR THREE PINNED ARCHES.

Example 12.—Figure 19 (a) shows an arch which is pinned at its supports A and B and at the crown C, the equation to the arch being :—

Height of arch above supports $= (40-0{\cdot}016x^2)$ feet where x is measured horizontally from the crown pin C in feet. Draw the influence line for horizontal thrust for this parabolic arch.

Also draw the influence lines for bending moment, radial shear, and normal thrust for the point P, which is 20 feet to the left of the crown pin C as shown in Fig. 19 (a).

Horizontal Thrust H.

Consider the 1 ton rolling load at a horizontal distance of x from the crown pin C and between A and C. Then the vertical

reaction at pin B $= \left(\dfrac{50-x}{100}\right)$ tons $= V_B$.

There is a pin at C and therefore there is no bending moment at this point. Equating the bending moments at C due to reaction

V_B and horizontal thrust H we have $\left(\dfrac{50-x}{100}\right) 50 = 40\,H.$

$$\therefore \quad H = \left(\frac{50-x}{80}\right) \text{ tons.}$$

This is a linear equation in x which shows that the influence line for horizontal thrust is a straight line between the pins at A and C.

When $x = 0$ (*i.e.*, load at crown pin C) then $H = 0{\cdot}625$ tons.
When $x = 50$ feet (*i.e.*, load at pin A) then H is zero.

Similarly, if the load is placed between C and B, a straight line would be obtained for the influence line. The required influence line is shown in Fig. 19 (c), and a system of rolling loads may be placed upon it in the usual way, so as to determine the maximum value of the horizontal thrust H at the supports.

Bending Moment at P.

By substituting $x = 20$ feet in the equation for the height of the arch, we find that the height of the arch at the point P is 33·6 ft. Now consider the 1 ton rolling load between P and the pin A at a horizontal distance of z from the point P. Then

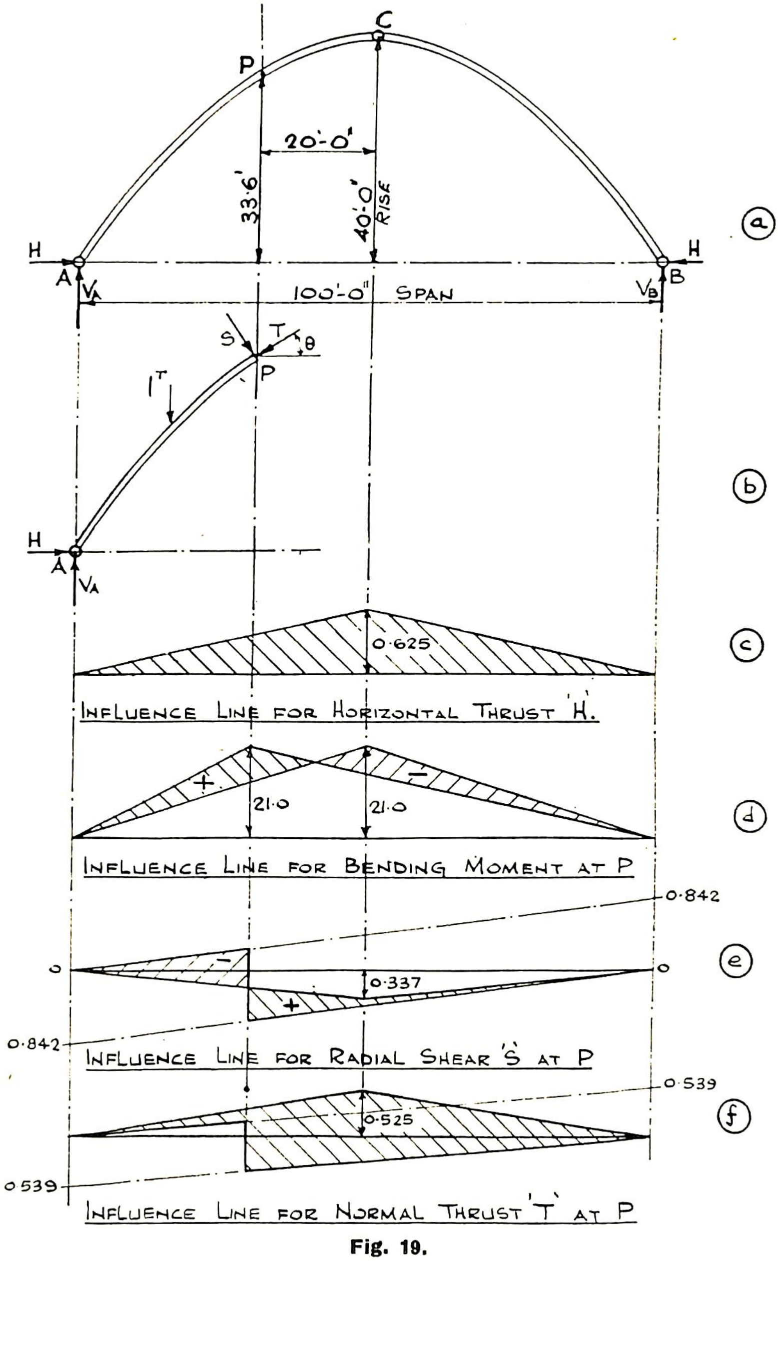

C
P
20'-0"
33·6'
40'-0" RISE
H
A
VA
100'-0" SPAN
VB
B
(a)
S
T
θ
P
1T
(b)
0·625
(c)
INFLUENCE LINE FOR HORIZONTAL THRUST 'H'.
+
−
21·0
21·0
(d)
INFLUENCE LINE FOR BENDING MOMENT AT P
0·842
0
0·337
0·842
(e)
INFLUENCE LINE FOR RADIAL SHEAR 'S' AT P
0·539
0·525
0·539
(f)
INFLUENCE LINE FOR NORMAL THRUST 'T' AT P

Fig. 19.

Bending moment at point P $= 33{\cdot}6\,H - 30\,V_A + 1z$ tons feet, and for this position of the load

$$V_A = \left(\frac{70+z}{100}\right) \text{ tons and } H = \left(\frac{30-z}{80}\right) \text{ tons.}$$

$$\text{Therefore bending moment} = 33{\cdot}6\left(\frac{30-z}{80}\right) - \left(\frac{210-7z}{10}\right) \text{tons feet.}$$

This equation is composed of two parts, both of which are linear equations in z. The first part is zero when z is 30 feet, and is a value of 33·6 (0·375) when z = zero. Therefore the first part of the influence line is 33·6 times the influence line for horizontal thrust H.

The second part is zero when z is 30 feet and rises up to its maximum value when z = zero, the maximum value being 21·0 tons vertically below the point P. The second part, therefore, is the bending moment influence line for point P when only vertical forces are considered (the simply supported bending moment influence line).

Similarly, if the 1 ton load is taken to the right of point P, it will be seen that the influence line is again composed of two parts, the first being 33·6 times the influence line for horizontal thrust, and the second being the simply supported bending moment influence line for the point P.

Therefore the final influence line encloses the difference of two triangles, as shown in Fig. 19 (d), the shaded areas being the relevant parts of the required influence line. It will be noticed that both triangles are of the same height. This is true for all points on a parabolic three-pinned arch, but it does not hold for semi-circular or segmental arches.

The part of the influence line marked positive, shows a bending moment which induces compression into the upper fibres of the arch.

Radial Shear (S) at P.

Fig. 19 (b) shows the arch cut short at point P, and the radial shear S and the normal thrust T are the forces which are set up in the arch, and which are necessary to give equilibrium to this part of the arch when carrying the 1 ton load.

Let θ be the angle of slope of the arch at the point P. Then equating forces we have

$$S = (V_A - 1)\cos\theta - H\sin\theta.$$

Therefore the influence line for radial shear is composed of two parts. The first part is the simply supported shear influence

line for the point P multiplied by cos θ, and the second part is the influence line for horizontal thrust multiplied by sin θ.

$$\text{Now } \tan\theta = 0{\cdot}64 \quad \therefore \quad \theta = 32^\circ\ 37'$$
$$\sin\theta = 0{\cdot}539 \text{ and } \cos\theta = 0{\cdot}842$$

The influence line for radial shear is therefore composed of a shear influence line, for which the ordinates at the supports are 0·842 minus the influence line for horizontal thrust multiplied by 0·539. The latter part encloses a triangle of maximum ordinate 0·337 at the centre of the span. The required influence line is shown in Fig. 19 (e), the negative sign indicating that the radial shear is outward from the arch on the relevant shaded areas.

Normal Thrust (T) at P.

Again equating forces for equilibrium in Fig. 19 (b) we have

$$\text{Normal thrust T} = (V_A - 1)\sin\theta + H\cos\theta$$

Therefore the influence line for normal thrust is the sum of two parts. The first part is the shear force influence line with ordinates of 0·539, and the second part is the influence line for horizontal thrust multiplied by 0·842, giving a maximum ordinate for the triangle of 0·525 at the centre of the span. The required influence line is shown in Fig. 19 (f).

Example 13.—Draw the influence lines for horizontal thrust, and for the bending moment, radial shear and normal thrust for the point P on the segmental arch rib shown in Fig. 20 (a).

Horizontal Thrust H.

The influence line encloses a triangle with a maximum ordinate of $\left(\dfrac{\text{Span}}{4 \times \text{Rise}}\right)$ = **1·21 tons** as shown in Fig. 20 (b).

Bending Moment at P.

The height of the arch at P is 17·82 feet. The influence line encloses two triangles as in the last example, but in this case they do not have the same value for their maximum ordinates.

The value of the horizontal thrust triangle ordinate is $(17{\cdot}82 \times 1{\cdot}21)$ = **21·6 tons,** and the value of the simply supported bending moment influence line ordinate is $\left(\dfrac{30 \times 70}{100}\right)$ = **21·0 tons** as in the last example. The required influence line is shown in Fig. 20 (c).

Radial Shear at P.

The angle of slope (θ) of the arch at the point P is 16° 26′.

$$\sin\theta = 0{\cdot}283. \qquad \cos\theta = 0{\cdot}959.$$

Similarly to the previous example this influence line is the difference between the simply supported shear force influence line multiplied by cos θ, and the influence line for horizontal thrust multiplied by sin θ.

The heights of the ordinates for the shear force influence line are **0·959 tons,** and the maximum ordinate for the horizontal thrust triangle is $(1{\cdot}21 \times 0{\cdot}283) = $ **0·343 tons.** The shape of the required influence line is shown in Fig. 20 (d), the negative sign indicating that the radial shear is outward from the centre O.

Normal Thrust at P.

Similarly to the previous example, this influence line is the sum of the simply supported shear force influence line multiplied by sin θ, and the influence line for horizontal thrust multiplied by cos θ.

The heights of the ordinates for the shear force influence line are **0·283 tons,** and the maximum ordinate for the horizontal thrust triangle is $(1{\cdot}21 \times 0{\cdot}959) = $ **1·16 tons.**

The shape of the required influence line is shown in Fig. 20 (e).

Example 14.—Draw the influence lines for the forces in the members KL, EF, KF and LF of the braced three-pinned arch shown in Fig. 21 (a). The arch is of 100 feet span, and the equation to the arch is height of arch above supports $= (40 - 0{\cdot}016\,x^2)$ feet, where x is measured horizontally from the crown pin C in feet. The rolling loads are carried by cross-girders at the upper panel points.

Force in KL.

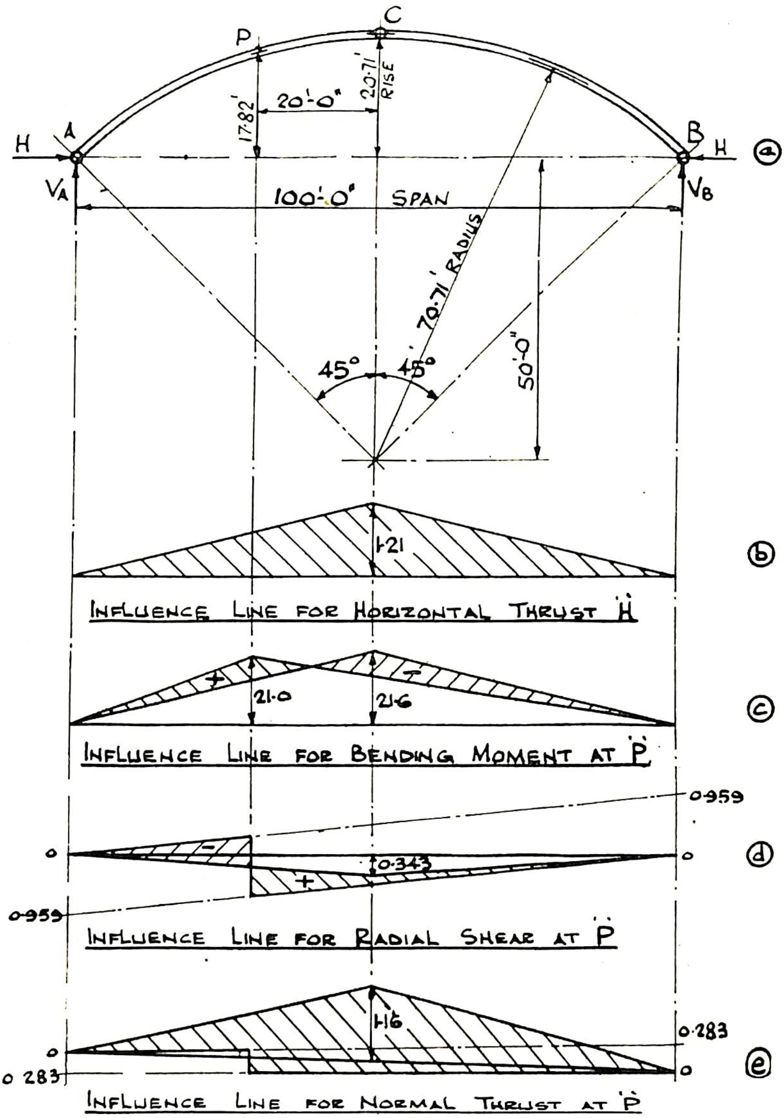
The force in member KL is the bending moment at the point F divided by the length of member LF

Consider the 1 ton rolling load to the right of point F. Then we have

$$\text{Force in member KL} = \left(\frac{33{\cdot}6\,H - 30\,V_A}{11{\cdot}4}\right) \text{tons.}$$

When this equation is positive the member KL is in tension, and when the equation is negative the member is in compression. Therefore it can be seen that the influence line required is composed of two parts :—

The first part encloses a triangle with a maximum ordinate at the centre of the span of value $\left(\frac{33{\cdot}6\,H}{11{\cdot}4}\right) = $ **1·84 tons,** since the horizontal thrust has a maximum value of 0·625 tons when the load is at the centre pin.



C
P
20·71' RISE
17·82'
20'-0"
A
B
H
H
(a)
V_A
V_B
100'-0" SPAN
70·71' RADIUS
50'-0"
45°
45°
1·21
(b)
INFLUENCE LINE FOR HORIZONTAL THRUST 'H'
+
−
21·0
21·6
(c)
INFLUENCE LINE FOR BENDING MOMENT AT 'P'
0·959
0
−
0·343
+
0
(d)
0·959
INFLUENCE LINE FOR RADIAL SHEAR AT 'P'
1·16
0·283
0
0
0 283
(e)
INFLUENCE LINE FOR NORMAL THRUST AT 'P'

Fig. 20.

The second part also encloses a triangle with a maximum ordinate vertically below point F of value

$$\left(\frac{30}{11{\cdot}4} \cdot V_A\right) = \textbf{1·84 tons.}$$

It will be seen that the height of the maximum ordinates of each of the triangles is the same. This was to be expected since in example 12, the heights of the influence line triangles were equal. This is only true for a parabolic arch.

The final shape of the influence line for the force in member KL is shown in Fig. 21 (b).

Force in EF.

The force in this member is the bending moment at the point K divided by the perpendicular distance from K to the member EF.

Now consider the 1 ton rolling load to the right of point K.

$$\text{Then force in member EF} = \left(\frac{45\,H - 20\,V_A}{15{\cdot}1}\right) \text{ tons.}$$

When this equation is positive the member EF is in compression, and when the equation is negative the member is in tension. It will again be seen that the influence line for the force in member EF is composed of two parts.

The first part encloses a triangle having a maximum ordinate at the centre of the span of value $\left(\frac{45\,H}{15{\cdot}1}\right)$ = **1·87 tons,**

The second part also encloses a triangle having a maximum ordinate vertically below K of value $\left(\frac{20\,V_A}{15{\cdot}1}\right)$ = **1·06 tons.**

The required influence line is shown in Fig. 21 (c).

Force in KF.

If a section is taken to cut the member KF and any two other members, then the other two members will intersect at point P, as shown in Fig. 21 (a).

Then the force in member KF is the bending moment at point P divided by the perpendicular distance from P to the member KF.

Now consider the 1 ton rolling load to the left of K at a distance of y from the support at A.

$$\text{Then force in member KF} = \left(\frac{45H - 55{\cdot}75\,(y/100)}{18{\cdot}3}\right) \text{ tons.}$$

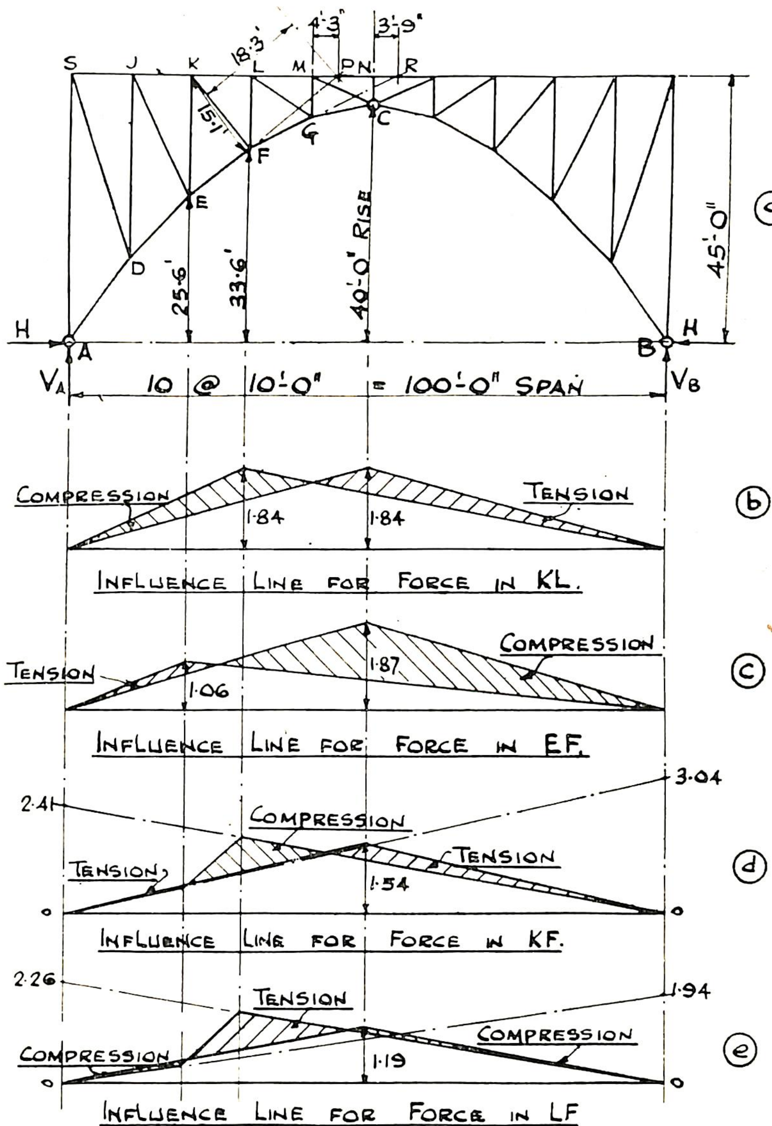

18.3'
4'3"
3'-9"
S
J
K
L
M
P
N
R
15.1'
G
C
F
E
D
25.6'
33.6'
40'-0" RISE
45'-0"
(a)
H
A
B
H
V_A
10 @ 10'-0" = 100'-0" SPAN
V_B
COMPRESSION
TENSION
1.84
1.84
(b)
INFLUENCE LINE FOR FORCE IN KL.
COMPRESSION
TENSION
1.87
1.06
(c)
INFLUENCE LINE FOR FORCE IN EF.
3.04
2.41
COMPRESSION
TENSION
TENSION
1.54
(d)
0
0
INFLUENCE LINE FOR FORCE IN KF.
2.26
1.94
TENSION
COMPRESSION
COMPRESSION
1.19
(e)
0
0
INFLUENCE LINE FOR FORCE IN LF

Fig. 21.

When this equation is positive, the member is in tension, and when the equation is negative, the member is in compression. The first part of this equation encloses a triangle with a minimum ordinate at the centre of the span of value **1·54 tons.**

The second part is a linear equation in y, which, although the actual part of the diagram is between A and K, can be produced to cut the right-hand reaction with an ordinate of

$$\left(\frac{55 \cdot 75}{18 \cdot 3}\right) = \mathbf{3 \cdot 04 \text{ tons.}}$$

Similarly, if the 1 ton load is taken to the right of L, the influence line may be considered as two separate parts. The first part will be a triangle of maximum ordinate **1·54 tons** at midspan, and the second part, although only considered between L and B, can be produced to cut the left-hand reaction with an ordinate of $\left(\frac{44 \cdot 25}{18 \cdot 3}\right) =$ **2·41 tons.** It has been seen previously that the influence line between any two cross-girders must be linear, and so the final shape of the influence line will be as shown in Fig, 21 (d).

It will be seen that the height of an ordinate on the influence line above a pinned support is the distance from this support to the point P, divided by the distance from P to the member in question.

Force in LF.

If a section is taken to cut the member LF and any two other members, the two other members will intersect at the point R as shown in Fig. 21 (a).

For the member LF, a similar procedure to that adopted for the member KF can be used, and a similar result is obtained.

The first part of the influence line is a triangle for which the equation is $\left(\frac{45 \text{ H}}{23 \cdot 75}\right)$ giving a maximum ordinate of **1·19 tons** at midspan.

The height of the ordinate erected below the left-hand reaction is $\left(\frac{53 \cdot 75}{23 \cdot 75}\right) =$ **2·26 tons,** and the ordinate erected below the right-hand reaction is $\left(\frac{46 \cdot 25}{23 \cdot 75}\right) =$ **1·94 tons.**

The final shape of the influence line is as shown in Fig. 21 (e).

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76. Shearing and Bearing Values for High Tensile Structural Steel Shop Rivets, in accordance with B.S.S. No. 548/1934.
77. Permissible Compressive Stresses for High Tensile Structural Steel, manufactured in accordance with B.S.S. 548/1934. Connected.
78. Velocity of Flow in Pipes for a Given Delivery. Connected.
79. Delivery of Water in Pipes for a Given Head.
80. (See No. 105).
81. Involute Toothed Gearing Chart.
82. Steam Pipe Design. Chart Showing Flow of Steam Through Pipes.
83. Variation of Suction Lift and Temperature for Centrifugal Pumps.
84. Nomograph for Uniformly Distributed Loads on British Standard Beams.
85. " " " " " Connected.
86. " " " " "
87. Notes on Beam Design and on Use of Data Sheets, Nos. 84-5-6.
88. " " " " "
89. Curve Relating Natural Frequency and Deflection.
90. Vibration Transmissibility Curved or Elastic Suspension. Connected.
91. Instructions and Examples in the Use of Data Sheets, Nos. 89 and 90.
92. Pressure on Sides of Bunker.
93-4-5-6-7. Rolled Steel Sections.
98-99-100. Boiler Safety Valves.
101. Nomograph Chart for Working Stresses in Mild Steel Columns.
102. Pressure Required for Blanking and Piercing.
103. Punch and Die Clearances for Blanking and Piercing.
104. Nomograph for Valley Angles of Hoppers and Chutes.
105. Permissible Working Stresses in Mild Steel Struts to B.S. 449, 1948.

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